Topic Z1 Vectors (Pre-TT B) [45] MARKSCHEME

1.

METHOD 1

$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c\ (=17)$	A1	For correct vector *OR* LHS of equation
$\theta = \sin^{-1} \dfrac{\lvert[1, 4, -1].[-7, 3, -1]\rvert}{\sqrt{1^2+4^2+1^2}\sqrt{7^2+3^2+1^2}}$	M1√	For using correct vectors for line and plane f.t. from normal
	M1*	For using scalar product of line and plane vectors
	M1	For calculating both moduli in denominator
$\theta = \sin^{-1} \dfrac{6}{\sqrt{18}\sqrt{59}} = 10.6°$	A1√ (*dep)	For scalar product. f.t. from their numerator
(10.609...°, 0.18517...)	A1 7	For correct angle

METHOD 2

$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c$	A1	For correct vector *OR* LHS of equation
$7x - 3y + z = 17$	M1√	For attempting to find RHS of equation f.t. from **n** or LHS of equation
$d = \dfrac{\lvert 21-12+2-17\rvert}{\sqrt{7^2+3^2+1^2}} = \dfrac{6}{\sqrt{59}}$	M1	For using distance formula from a point on the line, e.g. (3, 4, 2), to the plane
	A1√	For correct distance. f.t. from equation
$\theta = \sin^{-1} \dfrac{\frac{6}{\sqrt{59}}}{\sqrt{1^2+4^2+1^2}} = 10.6°$	M1	For using trigonometry
(10.609...°, 0.18517...)	A1	For correct angle

[7]

2.

(i)		$\begin{pmatrix}2\\3\\-1\end{pmatrix} \times \begin{pmatrix}4\\-1\\-1\end{pmatrix} = \begin{pmatrix}-4\\-2\\-14\end{pmatrix} = -2\begin{pmatrix}2\\1\\7\end{pmatrix}$	M1 A1	Or any multiple	
		$\begin{pmatrix}3\\0\\1\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}2\\-2\\0\end{pmatrix}$	B1	Or negative	Or use of n.(a₁ + pb₁ +kn) = n.(a₂ + qb₂) **B1** followed by attempt to calculate magnitude of kn **M1**
		shortest distance $= \dfrac{\left\lvert \begin{pmatrix}2\\-2\\0\end{pmatrix} \cdot \begin{pmatrix}2\\1\\7\end{pmatrix} \right\rvert}{\sqrt{2^2+1^2+7^2}} = \dfrac{2}{\sqrt{54}}$ oe	M1 A1 [5]	Component of their vector in their direction	
(ii)		$2x + y + 7z = \ldots$ $\ldots 11$	B1ft B1 dep [2]	ft from 4(i) only if 1st M1 mark gained If zero, then sc **1** for any correct **vector** equation.	

3.

4(a)	$\begin{vmatrix}1 & -3 & 2\\k & 1 & -1\\6 & -5 & k-1\end{vmatrix} = k-1-5+3(k(k-1)+6)+2(-5k-6)$	M1 A1	3.1a 1.1b
	$3k^2 - 12k = 0 \Rightarrow k = \ldots$	M1	3.1a
	$k = 0$ or 4	A1	1.1b
		(4)	

(b)(i)	$\begin{pmatrix}1 & -3 & 2\\ 5 & 1 & -1\\ 6 & -5 & 4\end{pmatrix}^{-1} = \frac{1}{15}\begin{pmatrix}-1 & 2 & 1\\ -26 & -8 & 11\\ -31 & -13 & 16\end{pmatrix}$	M1 A1	1.1b 1.1b
	$\frac{1}{15}\begin{pmatrix}-1 & 2 & 1\\ -26 & -8 & 11\\ -31 & -13 & 16\end{pmatrix}\begin{pmatrix}-7\\ -5\\ 1\end{pmatrix} = \ldots$	M1	1.1b
	$\left(-\frac{2}{15}, \frac{233}{15}, \frac{298}{15}\right)$	A1	1.1b
(b)(ii)	Three planes that meet at a point.	A1	2.2a
		(5)	

4.

7 (i) *EITHER*: (**AG** is **r** =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
Normal to *BCD* is	M1	For finding vector product of any two of $\pm[1, -4, -1], \pm[2, 1, 1], \pm[1, 5, 2]$
$\mathbf{n} = k[1, 1, -3]$	A1	For correct **n**
Equation of *BCD* is $\mathbf{r}.[1, 1, -3] = -6$	A1	For correct equation (or in cartesian form)
Intersect at $(6+t) + 4 + (-3)(8+t) = -6$	M1	For substituting point on *AG* into plane
$t = -4$ ($t = -1$ using $[3, 4, 5]$) $\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1	For correct position vector of *M* AG
OR: (**AG** is **r** =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
$\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$, where $\mathbf{u} = [2, 1, 3]$ *or* $[1, 5, 4]$ *or* $[3, 6, 5]$ $\mathbf{v}, \mathbf{w} =$ two of $[1, -4, -1], [1, 5, 2], [2, 1, 1]$	M1 A1	For a correct parametric equation of *BCD*
e.g. $(x =)\ 6+t = 2 + \lambda + \mu$ $(y =)\ 4 = 1 - 4\lambda + 5\mu$ $(z =)\ 8+t = 3 - \lambda + 2\mu$	M1	For forming 3 equations in t, λ, μ from line and plane, and attempting to solve them
$t = -4$ *or* $\lambda = -\frac{1}{3}, \mu = \frac{1}{3}$	A1	For correct value of t *or* λ, μ
$\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1 6	For correct position vector of *M* AG
(ii) *A, G, M* have $t = 0, -3, -4$ *OR* $AG = 3\sqrt{2}, AM = 4\sqrt{2}$ *OR* $\mathbf{AG} = [-3, 0, -3], \mathbf{AM} = [-4, 0, -4]$ $\Rightarrow AG : AM = 3 : 4$	B1 1	For correct ratio AEF
(iii) $\mathbf{OP} = \mathbf{OC} + \frac{4}{3}\mathbf{CG}$	M1	For using given ratio to find position vector of *P*
$= \left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right]$	A1 2	For correct vector
(iv) *EITHER*: Normal to *ABD* is	M1	For finding vector product of any two of $\pm[4, 3, 5], \pm[1, 5, 2], \pm[3, -2, 3]$
$\mathbf{n} = k[19, 3, -17]$	A1	For correct **n**
Equation of *ABD* is $\mathbf{r}.[19, 3, -17] = -10$	M1	For finding equation (or in cartesian form)
$19.\frac{11}{3} + 3.\frac{11}{3} - 17.\frac{16}{3} = -10$	A1	For verifying that *P* satisfies equation
OR: Equation of *ABD* is $\mathbf{r} = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$ (etc.)	M1	For finding equation in parametric form
$\left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right] = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$	M1	For substituting *P* and solving 2 equations for λ, μ
$\lambda = -\frac{2}{3}, \quad \mu = \frac{1}{3}$	A1	For correct λ, μ
	A1	For verifying 3rd equation is satisfied
OR: $\mathbf{AP} = \left[-\frac{7}{3}, -\frac{1}{3}, -\frac{8}{3}\right]$	M1 A1	For finding 3 relevant vectors in plane *ABDP* For correct **AP** *or* **BP** *or* **DP**
$\mathbf{AB} = [-4, -3, -5], \ \mathbf{AD} = [-3, 2, -3]$ $\Rightarrow \mathbf{AB} + \mathbf{AD} = [-7, -1, -8]$	M1	For finding **AB, AD** *or* **BA, BD** *or* **DB, DA**
$\Rightarrow \mathbf{AP} = \frac{1}{3}(\mathbf{AB} + \mathbf{AD})$	A1 4 13	For verifying linear relationship

5.

5(a)	$OC = 0.8\mathbf{k}$, $OB = 3\mathbf{i} + 0.8\mathbf{k}$ and $OD = 1.2\mathbf{j} + 1.5\mathbf{k}$, or $CB = 3\mathbf{i}$, and $CD = 1.2\mathbf{j} + 0.7\mathbf{k}$	B1	3.3
	So plane has equation **r** = their OC + their λCB + their μCD (oe) OR $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(3\mathbf{i}) = 0$ and $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(1.2\mathbf{j} + 0.7\mathbf{k}) = 0$ leading to *a* =..., *b* =... and *c* =... (may use vector product)	M1	1.1b
	Equation is $\mathbf{r} = 0.8\mathbf{k} + \lambda(3\mathbf{i}) + \mu(1.2\mathbf{j} + 0.7\mathbf{k})$ OR normal is $\mathbf{n} = p(7\mathbf{j} - 12\mathbf{k})$	A1	1.1b
	$x = 3\lambda$, $y = 1.2\mu$ and $z = 0.8 + 0.7\mu \Rightarrow 70y - 120z = -96$ OR $(0.8\mathbf{k}).(7\mathbf{j} - 12\mathbf{k}) = -9.6 \Rightarrow d = -9.6$	M1	1.1b
	Equation is $\mathbf{r}.(7\mathbf{j} - 12\mathbf{k}) = -9.6$ (or a multiple e.g. $\mathbf{r}.(70\mathbf{j} - 120\mathbf{k}) = -96$)	A1	2.5
(b)	Full attempt to find the minimum distance from the centre of the base rectangle to the plane – e.g. using the distance formula for closest point, or first finding the intersection point then finding the distance. Must have correct starting point (1.5,0.6,0).	M1	3.1b
	E.g. Minimum distance $= \dfrac{\lvert 0 \times 1.5 + 7 \times 0.6 + (-12) \times 0 + 9.6 \rvert}{\sqrt{0^2 + 7^2 + (-12)^2}} = \ldots$	M1	3.4
	= 0.993 m or 99.3 cm or 993 mm (to 3 s.f.) Accept awrt.	A1	1.1b
		(3)	
(c)	E.g. the boards will not have negligible thickness, which should be taken into account in the model, or wooden boards will bow and so not form planes.	B1	3.5b
		(1)	
5(a) Alt	Sets up equation of plane as $ax + by + c = d$	B1	3.3
	Identifies at least three points on the plane and substitutes in to the equation to form simultaneous equations. E.g. (3,0,0.8), (0,0,0.8), (0,1.2,1.5) and (3,1.2,1.5) give $3a + 0.8c = d$, $0.8c = d$, $1.2b + 1.5c = d$, $3a + 1.2b + 1.5c = d$. Note may use *d* = 1 with only 3 equations.	M1	1.1b
	Solves to find correct corresponding values. E.g. With *d* = 1, *c* = 1.25, *a* = 0 and *b* = $-\dfrac{35}{48}$ (accept appropriate multiples)	A1	1.1b
	Forms plane equation in correct form with their values. E.g. $-\dfrac{35}{48}y + \dfrac{5}{4}z = 1 (\Rightarrow 35y - 60z = -48) \Rightarrow \mathbf{r} \cdot \mathbf{n} = d$	M1	1.1b
	Equation is $\mathbf{r} \cdot (35\mathbf{i} - 60\mathbf{j}) = -48$ (or any multiple)	A1	2.5