

Topic Z2 Hyperbolics and further calculus (Post-TT A) [56]

1.

It is given that $f(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $f''(x) = \frac{1}{2(x+1)^2}$. [6]

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2 . [4]

2.

$$f(x) = \frac{1}{\sqrt{x^2 + 2x + 10}}$$

(a) Determine $\int f(x) dx$ [3]

(b) Hence show that the mean value of $f(x)$ over the interval $[2, 20]$ may be expressed in the form $a \ln(b + c\sqrt{2})$, where a , b and c are rational constants to be determined. [3]

3.

(a) Prove that $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} \equiv 2 \coth \theta$

Explicitly state any hyperbolic identities that you use within your proof.

[4 marks]

(b) Solve $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} = 4$ giving your answer in an exact form.

[2 marks]

4.

(i) Given that

$$y = x\sqrt{1-x^2} - \cos^{-1} x,$$

find $\frac{dy}{dx}$ in a simplified form. [4]

(ii) Hence, or otherwise, find the exact value of $\int_0^1 2\sqrt{1-x^2} dx$. [3]

5.

It is given that $f(x) = \frac{x^2 + 9x}{(x-1)(x^2 + 9)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Hence find $\int f(x) dx$. [2]

6.

Given that

$$\int_0^1 \frac{1}{\sqrt{16+9x^2}} dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}} dx = \ln a,$$

find the exact value of a .

[6]

7.

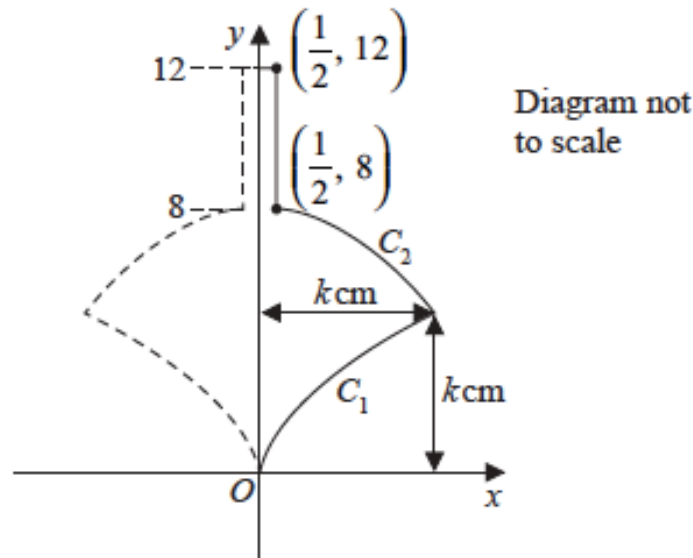


Figure 2

Figure 2 shows a sketch of the cross-section of a design for a child's spinning top. The top is formed by rotating the region bounded by the y -axis, the curve C_1 , the curve C_2 , the line with equation $x = \frac{1}{2}$ and the line with equation $y = 12$, through 360° about the y -axis.

The curve C_1 has equation

$$y = k^{\frac{2}{3}}x^{\frac{1}{3}} \quad 0 \leq x \leq k$$

and the curve C_2 has equation

$$y = \frac{32k^2 - k - (32 - 4k)x^2}{4k^2 - 1} \quad \frac{1}{2} \leq x \leq k$$

(a) Show that
$$\int_k^8 ((4k^2 - 1)y - (32k^2 - k)) dy = \frac{1}{2}(8 - k)(4k^3 - 32k^2 + k - 8)$$
 (3)

Hence find

(b) the value of k that gives the maximum value for the volume of the spinning top, (9)

(c) the maximum volume of the spinning top. (3)