

Topic Z2 Hyperbolics and further calculus (Post-TT B) [53]

1.

- (i) Using the definition of $\cosh x$ in terms of e^x and e^{-x} , prove that

$$\cosh 2x = 2 \cosh^2 x - 1. \quad [3]$$

- (ii) Hence solve the equation

$$\cosh 2x - 7 \cosh x = 3,$$

giving your answer in logarithmic form. [4]

(Total 7 marks)

2.

- (i) Write down and simplify the first three non-zero terms of the Maclaurin series for $\ln(1 + 3x)$.

[3]

- (ii) Hence find the first three non-zero terms of the Maclaurin series for

$$e^x \ln(1 + 3x),$$

simplifying the coefficients. [3]

(Total 6 marks)

3.

- (i) Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x} , prove that

$$\sinh 2x = 2 \sinh x \cosh x. \quad [4]$$

- (ii) Show that the curve with equation

$$y = \cosh 2x - 6 \sinh x$$

has just one stationary point, and find its x -coordinate in logarithmic form. Determine the nature of the stationary point. [8]

(Total 12 marks)

4.

- (i) Show that $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$. [2]

- (ii) Given that $y = \cosh(a \sinh^{-1} x)$, where a is a constant, show that

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - a^2 y = 0. \quad [5]$$

(Total 7 marks)

5.

By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5} dx.$$

[5]

(Total 5 marks)

6.

Evaluate the improper integral $\int_0^{\infty} \frac{4x - 30}{(x^2 + 5)(3x + 2)} dx$, showing the limiting process used.

Give your answer as a single term.

[8 marks]

(Total 8 marks)

7.

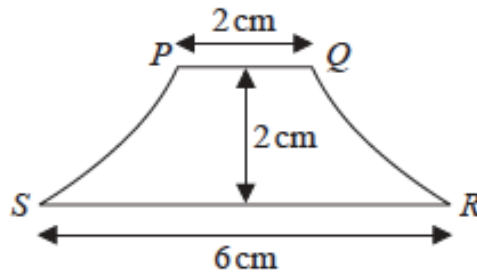


Figure 1

Figure 1 shows the central vertical cross section $PQRS$ of a handle for a drawer. Measurements were taken and the handle was found to have a height of 2 cm and the lengths of the straight lines PQ and SR were 2 cm and 6 cm respectively, as shown in Figure 1.

The handle is modelled by a solid of revolution of a curve C about the y -axis. The curve C has parametric equations

$$x = a \cos^2 \theta \quad y = b \tan \theta \quad \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$$

where a and b are constants.

Find, according to the model,

(a) (i) the value of a ,

(ii) the value of b .

(4)

(b) Hence, using calculus, determine the volume of the handle, according to the model.

(6)

(Total 10 marks)