Topic Z2 Hyperbolics and further calculus (Post-TT B) [53] MARKSCHEME

1.

(i) Correct definition of cosh x or cosh $2x$	B1	
Attempt to sub. in RHS and simplify	M1	or LHS if used
Clearly produce A.G.	A1	
(ii) Write as quadratic in cosh x	M1	($2\cosh^2 x - 7\cosh x - 4 = 0$)
Solve their quadratic accurately	A1√	Factorise/quadratic formula
Justify one answer only	B1	State cosh $x \geq 1$/graph; allow ≥ 0
Give ln(4 + √15)	A1	cao; any one of $\pm\ln(4 \pm \sqrt{15})$ or decimal equivalent of ln ( )

2.

(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$	M1	Allow e.g. $3x^2$, 2! etc.
	M1	Attempt to simplify $(3x)^2$ etc.
$= 3x - 9x^2/2 + 9x^3$	A1	cao
(ii) Produce ($1 + x + x^2/2$)	B1	
	M1	Mult. 2 reasonable attempts, each of 3 terms (non-zero)
Get $3x - 3x^2/2 + 6x^3$	A1√	From their series
	SC M1	Reasonable attempt at diff. and replace $x = 0$ (2 correct)
	M1√	Put their values into correct Maclaurin expansion
	A1	cao
		(Applies to either/both parts)

3.

(i) Correct def^n of cosh x and sinh x	B1,B1	
Expand $2.\frac{1}{2}(e^x - e^{-x}).\frac{1}{2}(e^x + e^{-x})$	M1	Reasonable attempt
Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G.	A1	
(ii) Attempt to diff. and solve $dy/dx = 0$	M1	Reasonable attempt
Use (ii) to get A cosh x (B sinh x+ C)=0	M1	
Clearly see cosh $x > 0$ or similar for one useable factor only	B1	
Attempt to solve sinh x = - C/B	M1	Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$	A1	
Justify one answer only for sinh x = -C/B	B1	
Accurate test for MINIMUM	B1	First or second diff^t test with numeric evidence
	B1	Correct value(s) for min.

4.

(i) Reasonable attempt to differentiate $\sinh y = x$ to get dy/dx in terms of y — M1 Allow $\pm\cosh y\, dy/dx = 1$

Replace $\sinh y$ to A.G. — A1 Clearly use $\cosh^2 - \sinh^2 = 1$

SC Attempt to diff. $y = \ln(x+\sqrt{(x^2+1)})$ using chain rule — M1

Clearly tidy to A.G. — A1

(ii) Reasonable attempt at chain rule — M1 To give a product

Get $dy/dx = a \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$ — A1

Reasonable attempt at product/quotient — M1 Must involve sinh and cosh

Get d^2y/dx^2 correctly in some form — A1√ From $dy/dx = k \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$

Substitute in and clearly get A.G. — A1

SC Write $\sqrt{(x^2+1)}dy/dx = k \sinh(a\sinh^{-1}x)$ or similar

Derive the A.G.

5.

$= \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{(2x-1)^2+4}\,dx$ *OR* $\frac{1}{4}\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{(x-\frac{1}{2})^2+1}\,dx$	B1	For correct denominator (in 2nd case must include $\frac{1}{4}$)
	M1	For integration to $k\tan^{-1}(ax+b)$ or $k\ln\left(\frac{ax+b-c}{ax+b+c}\right)$
$= \frac{1}{2}\left[\frac{1}{2}\tan^{-1}\frac{2x-1}{2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$ *OR* $\frac{1}{4}\left[\tan^{-1}\left(x-\frac{1}{2}\right)\right]_{\frac{1}{2}}^{\frac{3}{2}}$	A1	FT for $ax+b$ from their denominator For correct integration
$= \frac{1}{4}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{1}{16}\pi$	M1	For substituting limits in any $\tan^{-1}$ expression
	A1 [5]	For correct value

6.

Splits integrand into partial fractions of the form $\frac{Ax+B}{x^2+5}+\frac{C}{3x+2}$	AO3.1a	M1	$\frac{4x-30}{(x^2+5)(3x+2)} \equiv \frac{Ax+B}{x^2+5}+\frac{C}{3x+2}$ $\Rightarrow 4x-30 \equiv (Ax+B)(3x+2)+C(x^2+5)$
Sets up an identity from which to solve for A, B and C	AO1.1a	M1	Compare coefficients of x: $4 = 2A+3B$ Compare coeffcients of x^2: $0 = 3A+C$
Obtains correct values of A, B and C CAO	AO1.1b	A1	Compare constant terms $-30 = 2B+5C$ $\therefore -30 = 2B-15A$
Integrates 'their' two terms correctly FT provided both M1 marks awarded	AO1.1b	A1F	$-30 = 2B-15(\frac{4-3B}{2})$ $B = 0$ $A = 2$ $C = -6$

Applies the laws of logs to 'their' integral correctly	AO1.1a	M1	$\int \frac{4x-30}{(x^2+5)(3x+2)}dx = \int \frac{2x}{x^2+5} - \frac{6}{3x+2}dx$ $= \ln(x^2+5) - 2\ln(3x+2) + c$
Applies limits (a and 0) to 'their' integral correctly	AO1.1a	M1	$\int_0^\infty \frac{4x-30}{(x^2+5)(3x+2)}dx$ $= \lim_{a\to\infty} \int_0^a \frac{2x}{x^2+5} - \frac{6}{3x+2}dx$
Shows the limiting process used with clear detailed working	AO2.1	R1	$= \lim_{a\to\infty} \left[\ln(x^2+5) - 2\ln(3x+2)\right]_0^a$
Obtains correct single term solution CAO	AO1.1b	A1	$= \lim_{a\to\infty} \left[\ln\left(\frac{x^2+5}{(3x+2)^2}\right)\right]_0^a$ $= \lim_{a\to\infty} \left[\ln\left(\frac{a^2+5}{9a^2+12x+4}\right) - \ln\left(\frac{5}{4}\right)\right]$ $= \ln\left(\frac{1}{9}\right) - \ln\left(\frac{5}{4}\right)$ $= \ln\left(\frac{4}{45}\right)$
Total		**8**	

7.

2(a)(i)	$a\cos^2\frac{\pi}{3} = 1 \Rightarrow a = \ldots$ or $a\cos^2\frac{\pi}{6} = 3 \Rightarrow a = \ldots$	M1	3.4
	$a = 4$	A1	2.2b
(ii)	$b\tan\frac{\pi}{3} - b\tan\frac{\pi}{6} = 2 \Rightarrow b = \ldots$	M1	3.4
	$b = \sqrt{3}$	A1	2.2b
		(4)	
(b)	$V = \pi\int x^2 dy = \pi\int 16\cos^4\theta \times \sqrt{3}\sec^2\theta\, d\theta$	M1	3.4
	$= 16\pi\sqrt{3}\int \cos^2\theta\, d\theta$	A1	1.1b
	$= 16\pi\sqrt{3}\int \frac{\cos 2\theta + 1}{2} d\theta$	M1	3.1a
	$= 8\pi\sqrt{3}\left[\frac{1}{2}\sin 2\theta + \theta\right]$	A1	1.1b
	$= 8\pi\sqrt{3}\left[\frac{1}{2}\sin 2\theta + \theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = 8\pi\sqrt{3}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{6}\right)$	M1	3.4
	$= \frac{4\pi^2\sqrt{3}}{3} = 22.8\text{ cm}^3$	A1	1.1b
		(6)	