Topic Z2 Hyperbolics and further calculus (Pre-TT A) [49]

1.

(a) Find

$$\int \frac{1}{x^2 + 6x + 25} \mathrm{d}x$$

(3)

(b) Hence find the exact value of

$$\int_{-3}^{1} \left(1 - \frac{25}{x^2 + 6x + 25}\right) \mathrm{d}x$$

giving the answer in simplest form.

(3)

A student claims that the magnitude of the answer to part (b) gives the total area bounded by the curve $y = 1 - \frac{25}{x^2 + 6x + 25}$ and the x-axis between the line $x = -3$ and the line $x = 1$

(c) State, with a reason, whether or not the student is correct.

(1)

(Total 7 marks)

2.

(a) Using the definition of $\cosh x$ in terms of exponentials, prove that

$$4\cosh^3 x - 3\cosh x \equiv \cosh 3x$$

(3)

(b) Hence, or otherwise, solve the equation

$$\cosh 3x = 9\cosh x$$

(4)

(Total 7 marks)

3.

Given that

$$\mathrm{f}(x) = \mathrm{e}^{2x}\cos x$$

(a) Show that

$$\mathrm{f}''(x) = p\mathrm{f}(x) + q\mathrm{f}'(x)$$

where p and q are integers to be determined.

(5)

(b) Hence find the Maclaurin series for $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^5, giving each coefficient in its simplest form.

(3)

(Total 8 marks)

4.

The curve, C, has equation $y = \dfrac{x}{\cosh x}$

(a) Show that the x-coordinates of any stationary points of C satisfy the equation $\tanh x = \dfrac{1}{x}$

[3 marks]

(b) (i) Sketch the graphs of $y = \tanh x$ and $y = \dfrac{1}{x}$ on the axes below.

[2 marks]

(b) (ii) Hence determine the number of stationary points of the curve C.

[1 mark]

(c) Show that $\dfrac{d^2y}{dx^2} + y = 0$ at each of the stationary points of the curve C.

[4 marks]

(Total 10 marks)

5.

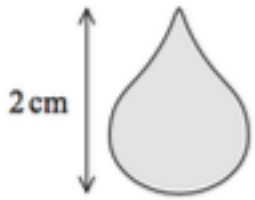

Figure 2

Figure 2 shows the image of a gold pendant which has height 2 cm. The pendant is modelled by a solid of revolution of a curve C about the y-axis. The curve C has parametric equations

$$x = \cos\theta + \frac{1}{2}\sin 2\theta, \quad y = -(1 + \sin\theta) \quad 0 \leqslant \theta \leqslant 2\pi$$

(a) Show that a Cartesian equation of the curve C is

$$x^2 = -(y^4 + 2y^3)$$

(4)

(b) Hence, using the model, find, in cm^3, the volume of the pendant.

(4)

(Total 8 marks)

6.

$$f(x) = \frac{x+2}{x^2+9}$$

(a) Show that

$$\int f(x)\mathrm{d}x = A\ln(x^2+9) + B\arctan\left(\frac{x}{3}\right) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence show that the mean value of $f(x)$ over the interval $[0, 3]$ is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval $[0, 3]$, of

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, where p and q are constants and q is in terms of k.

(2)

(Total 9 marks)