Topic Z2 Hyperbolics and further calculus (Pre-TT A) [49] MARKSCHEME

1.

1(a)	$\int \frac{1}{x^2+6x+25}\,dx = \int \frac{1}{(x+3)^2-9+25}\,dx = \int \frac{1}{(x+3)^2+16}\,dx$ or reaches integral in ϑ if using substitution.	M1	3.1a
	$= k\arctan\left(\frac{x+b}{a}\right)(+c)$ (or $k\vartheta$ where $4\tan\vartheta = x + 3$)	M1	1.1b
	$= \frac{1}{4}\arctan\left(\frac{x+3}{4}\right)+c$	A1	1.1b
(b)	$\int_{-3}^{1}\left(1-\frac{25}{x^2+6x+25}\right)dx = \left[x-\frac{25}{4}\arctan\left(\frac{x+3}{4}\right)\right]_{-3}^{1} = (1-...)-(-3-...)$	M1	1.1b
	$=\left(1-\frac{25}{4}\arctan\left(\frac{4}{4}\right)\right)-\left(-3-\frac{25}{4}\arctan 0\right)$	A1ft	1.1b
	$=4-\frac{25\pi}{16}$	A1	2.1
(c)	Since the graph crosses the *x*-axis at *x* = 0 the area lies partially below the *x*-axis, hence the expression does not give the total area as the part below the axis counts as negative which cancels the positive area, so the student is not correct.	B1	2.3

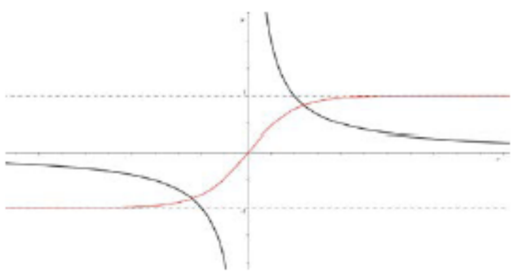
2.

7(a)	$4\cosh^3 x - 3\cosh x \equiv 4\left(\frac{e^x+e^{-x}}{2}\right)^3 - 3\left(\frac{e^x+e^{-x}}{2}\right)$	M1	1.2
	$\equiv 4\left(\frac{e^{3x}+3e^x+3e^{-x}+e^{-3x}}{8}\right)-3\left(\frac{e^x+e^{-x}}{2}\right)$	M1	1.1b
	$=\frac{e^{3x}}{2}+\frac{3e^x}{2}+\frac{3e^{-x}}{2}+\frac{e^{-3x}}{2}-\frac{3e^x}{2}-\frac{3e^{-x}}{2} \equiv \frac{e^{3x}+e^{-3x}}{2} \equiv \cosh 3x$ *	A1*	2.1
		(3)	
(b)	$\cosh 3x = 9\cosh x \Rightarrow 4\cosh^3 x - 3\cosh x = 9\cosh x$ $\cosh^2 x = 3$	M1	3.1a
	$\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3}+\sqrt{\left(\sqrt{3}\right)^2-1}\right)$	M1	1.1b
	$x=\ln\left(\sqrt{3}+\sqrt{2}\right)$ **or** $x=\ln\left(\sqrt{3}-\sqrt{2}\right)$	A1	1.1b
	$x=\ln\left(\sqrt{3}+\sqrt{2}\right)$ **and** $x=\ln\left(\sqrt{3}-\sqrt{2}\right)$ With no "solutions" being found by attempts to solve $\cosh x = 0$ or $\cosh x = -\sqrt{3}$	A1	2.3
		(4)	
			(7 marks)

3.

1(a)	$\mathrm{f}(x)=\mathrm{e}^{2x}\cos x \Rightarrow \mathrm{f}'(x)=2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x$	M1	1.1a
	$\mathrm{f}''(x)=4\mathrm{e}^{2x}\cos x-2\mathrm{e}^{2x}\sin x-\left(2\mathrm{e}^{2x}\sin x+\mathrm{e}^{2x}\cos x\right)$	M1 A1	1.1b 1.1b
	$\mathrm{f}''(x)=3\mathrm{e}^{2x}\cos x-4\mathrm{e}^{2x}\sin x=p\mathrm{e}^{2x}\cos x+q\left(2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x\right)$ $\Rightarrow p=\ldots,q=\ldots$	M1	3.1a
	$\mathrm{f}''(x)=-5\mathrm{f}(x)+4\mathrm{f}'(x)$	A1	2.1
		(5)	
(b)	$\mathrm{f}(0)=1,\mathrm{f}'(0)=2,\mathrm{f}''(0)=3,\mathrm{f}'''(0)=2,\mathrm{f}''''(0)=-7,\mathrm{f}^{v}(0)=-38$	M1	1.1b
	$\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\ldots$	M1	1.1b
	$\mathrm{f}(x)\approx 1+2x+\frac{3x^2}{2}+\frac{x^3}{3}-\frac{7x^4}{24}-\frac{19x^5}{60}$	A1	2.2a
		(3)	

4.

(a)	Uses quotient or product rule to obtain correct derivative	AO1.1b	B1	$y=\frac{x}{\cosh x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cosh x-x\sinh x}{\cosh^2 x}$
	Clearly sets 'their' $\frac{\mathrm{d}y}{\mathrm{d}x}$ numerator equal to 0	AO2.4	R1	Stationary point $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow\frac{\cosh x-x\sinh x}{\cosh^2 x}=0$ $\Rightarrow\cosh x-x\sinh x=0$
	Rearranges to complete a rigorous argument to show the required result. **AG**	AO2.1	R1	$\Rightarrow\frac{\sinh x}{\cosh x}=\frac{1}{x}$ $\Rightarrow\tanh x=\frac{1}{x}$ **AG**
(b)(i)	Sketches $\tanh x$ correctly including asymptotes	AO1.2	B1	
	Sketches $\frac{1}{x}$ correctly	AO1.2	B1	
(ii)	Deduces correct number of stationary points FT 'their' sketch in **(b)(i)**	AO2.2a	B1F	2 stationary points

(c)	Finds the second derivative	AO1.1a	M1	$\frac{d^2y}{dx^2}=\frac{\cosh^2 x(\sinh x - x\cosh x - \sinh x)}{\cosh^4 x} - \frac{2\cosh x \sinh x(\cosh x - x\sinh x)}{\cosh^4 x}$
	Obtains a correct expression for the second derivative	AO1.1b	A1	
	Deduces that the second term is zero by using results from part **(a)**	AO2.2a	R1	second term is zero at stationary points $\frac{d^2y}{dx^2}=-\frac{x}{\cosh x}=-y$
	Completes a rigorous argument to show the required result. AG Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	$\Rightarrow \frac{d^2y}{dx^2}+y=0$ **AG**
	Total		**10**	

5.

(a)	$x=\cos\theta+\sin\theta\cos\theta=-y\cos\theta$	M1	2.1
	$\sin\theta=-y-1$	M1	2.1
	$\left(\frac{x}{-y}\right)^2=1-(-y-1)^2$	M1	2.1
	$x^2=-\left(y^4+2y^3\right)$*	A1*	1.1b
		(4)	
b)	$V=\pi\int x^2\,dy=\pi\int -\left(y^4+2y^3\right)\,dy$	M1	3.4
	$=\pi\left[-\left(\frac{y^5}{5}+\frac{y^4}{2}\right)\right]$	A1	1.1b
	$=-\pi\left[\left(\frac{(0)^5}{5}+\frac{(0)^4}{2}\right)-\left(\frac{(-2)^5}{5}+\frac{(-2)^4}{2}\right)\right]$	M1	3.4
	$=1.6\pi\text{ cm}^3$ **or** awrt 5.03 cm^3	A1	1.1b
		(4)	
			(8 marks)

6.

(a)	$\mathrm{f}(x)=\frac{x+2}{x^2+9}=\frac{x}{x^2+9}+\frac{2}{x^2+9}$	B1	3.1a
	$\int\frac{x}{x^2+9}\,\mathrm{d}x=k\ln\left(x^2+9\right)(+c)$	M1	1.1b
	$\int\frac{2}{x^2+9}\,\mathrm{d}x=k\arctan\left(\frac{x}{3}\right)(+c)$	M1	1.1b
	$\int\frac{x+2}{x^2+9}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+c$	A1	1.1b
		(4)	
b)	$\int_0^3\mathrm{f}(x)\,\mathrm{d}x=\left[\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$ $=\frac{1}{2}\ln 18+\frac{2}{3}\arctan\left(\frac{3}{3}\right)-\left(\frac{1}{2}\ln 9+\frac{2}{3}\arctan(0)\right)$ $=\frac{1}{2}\ln\frac{18}{9}+\frac{2}{3}\arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value $=\frac{1}{3-0}\left(\frac{1}{2}\ln 2+\frac{\pi}{6}\right)$	M1	2.1
	$\frac{1}{6}\ln 2+\frac{1}{18}\pi$ *	A1*	2.2a
		(3)	
(c)	$\frac{1}{6}\ln 2+\frac{1}{18}\pi+\ln k$	M1	2.2a
	$\frac{1}{6}\ln 2k^6+\frac{1}{18}\pi$	A1	1.1b
		(2)	
			(9 marks)