Topic Z2 Hyperbolics and further calculus (Pre-TT B) [47]

1.

Solve the equation

$$6\cosh 2x + 4\sinh x = 7$$

giving your answers as exact logarithms.

(6)

(Total 6 marks)

2.

(i) By means of a suitable substitution, show that

$$\int \frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x$$

can be transformed to $\int \cosh^2\theta\,\mathrm{d}\theta$. [2]

(ii) Hence show that $\int \frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x = \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\cosh^{-1}x + c$. [4]

(Total 6 marks)

3.

The curves $y = \cos^{-1}x$ and $y = \tan^{-1}(\sqrt{2}x)$ intersect at a point A.

(i) Verify that the coordinates of A are $\left(\frac{1}{\sqrt{2}}, \frac{1}{4}\pi\right)$. [2]

(ii) Determine whether the tangents to the curves at A are perpendicular. [4]

(Total 6 marks)

4.

$$\mathrm{f}(x) = \begin{cases} \dfrac{kx}{x^2+6} & \text{for } 0 \leqslant x < 3 \\ \dfrac{k}{x^2-4} & \text{for } 3 \leqslant x \end{cases}$$

where k is a positive constant.

The area between the curve $y = \mathrm{f}(x)$ and the positive x-axis is $\frac{1}{4}$

Show that

$$k = \frac{1}{\ln a}$$

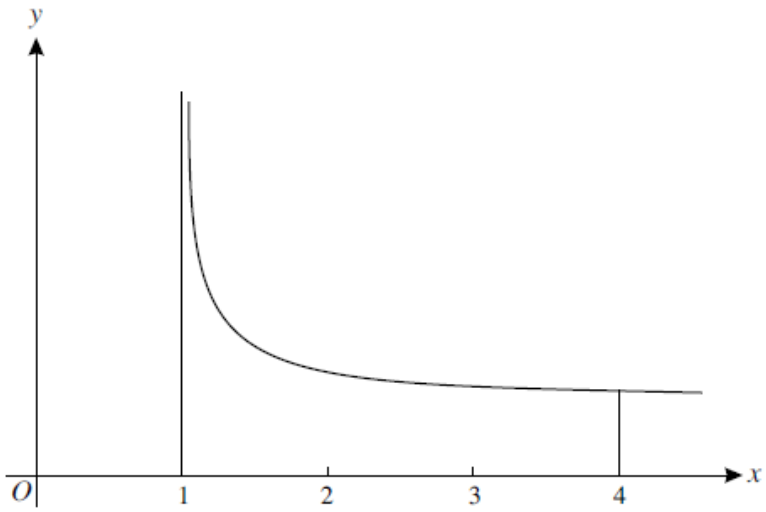
where a is a constant to be determined.

(8)

(Total 8 marks)

5.

(i) Use the substitution $x = \cosh^2 u$ to find $\int \sqrt{\frac{x}{x-1}}\,dx$, giving your answer in the form $f(x) + \ln(g(x))$. **[7]**



(ii) Hence calculate the exact area of the region between the curve $y = \sqrt{\frac{x}{x-1}}$, the x-axis and the lines $x = 1$ and $x = 4$ (see diagram). **[1]**

(iii) What can you say about the volume of the solid of revolution obtained when the region defined in part (ii) is rotated completely about the x-axis? Justify your answer. **[3]**

(Total 11 marks)

6.

$$y = \sin x \sinh x$$

(a) Show that $\frac{d^4y}{dx^4} = -4y$

(4)

(b) Hence find the first three non-zero terms of the Maclaurin series for y, giving each coefficient in its simplest form.

(4)

(c) Find an expression for the nth non-zero term of the Maclaurin series for y.

(2)

(Total 10 marks)