Topic Z2 Hyperbolics and further calculus (Pre-TT B) [47] MARKSCHEME

1.

Question	Scheme	Marks	AOs
1	$6(1+2\sinh^2 x)+4\sinh x=7$ and rearranges to quadratic form OR substitutes correct exponential identifies and rearranges to quartic in e^x, $\cosh 2x=\frac{e^{2x}+e^{-2x}}{2}$ and $\sinh x=\frac{e^x-e^{-x}}{2}$ used.	M1	3.1a
	$12\sinh^2 x+4\sinh x-1=0$ OR $3e^{4x}+2e^{3x}-7e^{2x}-2e^x+3=0$	A1	1.1b
	$(6\sinh x-1)(2\sinh x+1)=0\Rightarrow\sinh x=$ OR $(e^{2x}+e^x-1)(3e^{2x}-e^x-3)=0\Rightarrow e^x=$.	M1	1.1b
	$\sinh x=\frac{1}{6}$ or $\sinh x=-\frac{1}{2}$ OR $e^x=\frac{-1\pm\sqrt{5}}{2}$ or $e^x=\frac{1\pm\sqrt{37}}{6}$	A1	1.1b
	$x=\ln\left(a+\sqrt{1+a^2}\right)$ where *a* is one of their sinh *x* values OR undoes exponentials using ln	M1	1.2
	$x=\ln\left(\frac{1+\sqrt{37}}{6}\right)$, $x=\ln\left(\frac{-1+\sqrt{5}}{2}\right)$	A1	1.1b
		(6)	
			(6 marks)

2.

(i)	Let $x=\cosh\theta$ such that $dx=\sinh\theta\, d\theta$	M1		
	Clearly use $\cosh^2-\sinh^2=1$	A1	Clearly derive A.G.	
(ii)	Replace $\cosh^2\theta$	M1	Allow $a(\cosh 2\theta\pm 1)$	
	Attempt to integrate their expression	M1	Allow $b\sinh 2\theta\pm a\theta$	
	Get $\frac{1}{4}\sinh 2\theta+\frac{1}{2}\theta\ (+c)$	A1		
	Clearly replace for x to A.G.	B1	Condone no $+c$	
			SC Use expo. def^n; three terms	M1
			Attempt to integrate	M1
			Get $\frac{1}{8}(e^{2\theta}-e^{-2\theta})+\frac{1}{2}\theta\ (+c)$	A1
			Clearly replace for x to A.G.	B1

3.

4	**(i)**		For 1st curve $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$ For 2nd curve $\tan^{-1}\left(\sqrt{2}\cdot\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$	**B1** **B1** **[2]**		Alt: M1 Set up quadratic in sin or cos and solve A1 Both values correct
	(ii)		For 1st curve $y=\cos^{-1}x$, $\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$ For 2nd curve $y=\tan^{-1}x$, $\frac{dy}{dx}=\frac{\sqrt{2}}{1+2x^2}$ For 1st curve, when $x=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}}=\frac{-1}{\frac{1}{\sqrt{2}}}=-\sqrt{2}$ For 2nd curve, when $x=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$ Since $m_1\cdot m_2=-1$ then Yes	**B1** **B1** **M1** **A1** **[4]**	soi soi Substituting value into *their* derivatives and using $m_1 \times m_2 = ($ □ finding the product of gradients) Depends on exact correct numerical values being seen	Acceptable reason: One the negative reciprocal of the other. Condone: One the negative inverse of the other

4.

Question	Scheme	Marks	AOs
4	$\int_0^3 \frac{kx}{x^2+6}\,dx+\int_3^\infty \frac{k}{x^2-4}\,dx=\frac{1}{4}\Rightarrow k\left([\ldots]_0^3+[\ldots]_3^\infty\right)=\frac{1}{4}\Rightarrow k=\ldots$	M1	3.1a
	$(k)\int_0^3 \frac{x}{x^2+6}\,dx=(k)\left[\frac{1}{2}\ln\left(x^2+6\right)\right]_0^3=(k)\left(\frac{1}{2}\ln(15)-\frac{1}{2}\ln(6)\right)$	M1 A1	1.1b 1.1b
	$(k)\int \frac{1}{x^2-4}\,dx=(k)\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)$	M1	1.1b
	$\int_3^\infty \frac{1}{x^2-4}\,dx=\lim_{t\to\infty}\left[\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)\right]_3^t$ $=\frac{1}{4}\left(\lim_{t\to\infty}\ln\left(\frac{t-2}{t+2}\right)-\ln\left(\frac{1}{5}\right)\right)=\frac{1}{4}\ln 5$	M1 A1	3.1a 1.1b
	So $\frac{k}{2}\ln\left(\frac{15}{6}\right)+\frac{k}{4}\ln 5=\frac{1}{4}\Rightarrow k\ln\left(\frac{5}{2}\right)^2+k\ln(5)=1$ so $k=\frac{1}{\ln\left(\frac{125}{4}\right)}$	M1 A1	2.1 1.1b
		(8)	
			(8 marks)

5.

8(i)	$x=\cosh^2 u \Rightarrow \mathrm{d}u = 2\cosh u \sinh u \,\mathrm{d}u$	**B1**	For correct result
	$\int\sqrt{\frac{x}{x-1}}\,\mathrm{d}x = \int \frac{\cosh u}{\sinh u} 2\cosh u \sinh u\,\mathrm{d}u$	**M1**	For substituting throughout for x
	$=\int 2\cosh^2 u\,\mathrm{d}u$	**A1**	For correct simplified u integral
	$=\int(\cosh 2u+1)\mathrm{d}u = \sinh u\cosh u + u$	**M1**	For attempt to integrate $\cosh^2 u$
		A1	For correct integration
	$=x^{\frac{1}{2}}(x-1)^{\frac{1}{2}}+\ln\left(x^{\frac{1}{2}}+(x-1)^{\frac{1}{2}}\right)\ (+c)$	**M1**	For substituting for u
		A1 **7**	For correct result **oe** as $f(x)+\ln(g(x))$
(ii)	$2\sqrt{3}+\ln(2+\sqrt{3})$	**B1** **1**	
(iii)	$V=(\pi)\int_1^4 \frac{x}{x-1}\,\mathrm{d}x = (\pi)\left[x+\ln(x-1)\right]_1^4$	**M1**	For attempt to find $\int\frac{x}{x-1}\,\mathrm{d}x$
		A1	For correct integration (ignore π)
	$V\to\infty$	**B1** **3**	For statement that volume is infinite (independent of M mark)

6.

(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\sin x\cosh x+\cos x\sinh x$	M1	1.1a
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\cos x\cosh x+\sin x\sinh x+\cos x\cosh x-\sin x\sinh x$ $(=2\cos x\cosh x)$	M1	1.1b
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\cos x\sinh x-2\sin x\cosh x$	M1	1.1b
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=-4\sinh x\sin x=-4y$ *	A1*	2.1
		(4)	
b)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0=2,\ \left(\frac{\mathrm{d}^6y}{\mathrm{d}x^6}\right)_0=-8,\ \left(\frac{\mathrm{d}^{10}y}{\mathrm{d}x^{10}}\right)_0=32$	B1	3.1a
	Uses $y=y_0+xy_0'+\frac{x^2}{2!}y_0''+\frac{x^3}{3!}y_0'''+\ldots$ with their values	M1	1.1b
	$=\frac{x^2}{2!}(2)+\frac{x^6}{6!}(-8)+\frac{x^{10}}{10!}(32)$	A1	1.1b
	$=x^2-\frac{x^6}{90}+\frac{x^{10}}{113400}$	A1	1.1b
		(4)	
c)	$2(-4)^{n-1}\frac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
			(10 marks)