Topic Z2 Hyperbolics and further calculus (Pre-TT C) [47]

1.

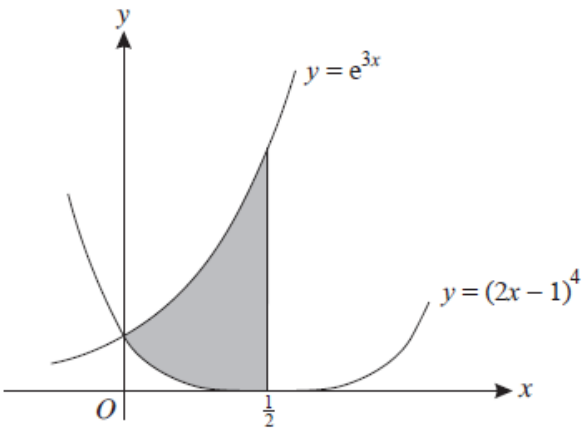
By first completing the square, find $\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,dx$, giving your answer in an exact logarithmic form. [6]

2.

(i) Express $\frac{4}{(1-x)(1+x)(1+x^2)}$ in partial fractions. [5]

(ii) Show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4}\,dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{1}{3}\pi$. [4]

3.



The diagram shows the curves $y = e^{3x}$ and $y = (2x-1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced. [9]

4.

(i) By first expanding $(e^x + e^{-x})^3$, or otherwise, show that $\cosh 3x \equiv 4\cosh^3 x - 3\cosh x$. [4]

(ii) Solve the equation $\cosh 3x = 6\cosh x$, giving your answers in exact logarithmic form. [5]

5.

$$y = \arctan(\sinh(x))$$

(a) Show that $\frac{d^3y}{dx^3} = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$

(7)

(b) Hence find $\frac{d^5y}{dx^5}$ in terms of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

(4)

(c) Find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^5

(3)