

Topic Z3 Differential equations (Post-TT B) [50]

1.

- (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = \sin x. \quad [6]$$

- (ii) Find the solution of the differential equation for which $y = 0$ and $\frac{dy}{dx} = \frac{4}{3}$ when $x = 0$. [4]

(Total 10 marks)

2.

Find the particular solution of the differential equation

$$x \frac{dy}{dx} + 3y = x^2 + x$$

for which $y = 1$ when $x = 1$, giving y in terms of x .

[8]

(Total 8 marks)

3.

The variables x and y satisfy the differential equation

$$2 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - 2y = 5e^{-2x}.$$

- (i) Find the complementary function of the differential equation. [2]
- (ii) Given that there is a particular integral of the form $y = pxe^{-2x}$, find the constant p . [4]
- (iii) Find the solution of the equation for which $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$. [5]

(Total 11 marks)

4.

- (i) Find the general solution of the differential equation

$$\frac{dy}{dx} + y \tan x = \cos^3 x,$$

expressing y in terms of x in your answer.

[8]

- (ii) Find the particular solution for which $y = 2$ when $x = \pi$. [2]

(Total 10 marks)

5.

The differential equation $\frac{d^2y}{dx^2} + 4y = \sin kx$ is to be solved, where k is a constant.

- (i) In the case $k = 2$, by using a particular integral of the form $ax \cos 2x + bx \sin 2x$, find the general solution. [7]
- (ii) Describe briefly the behaviour of y when $x \rightarrow \infty$. [2]
- (iii) In the case $k \neq 2$, explain whether y would exhibit the same behaviour as in part (ii) when $x \rightarrow \infty$. [2]

(Total 11 marks)