Topic Z3 Differential equations (Post-TT B) [50] MARKSCHEME

1.

(i) $\left(m^2+4=0\Rightarrow\right) m=\pm 2\mathrm{i}$	B1	For correct solutions of auxiliary equation (may be implied by correct CF)
CF = $A\cos 2x + B\sin 2x$	B1	For correct CF (**AEtrig** but not $Ae^{2\mathrm{i}x}+Be^{-2\mathrm{i}x}$ only)
PI = $p\sin x\ (+\,q\cos x)$	B1	State a trial PI with at least $p\sin x$
$-p\sin x\ (-q\cos x)+4p\sin x\ (+4q\cos x) = \sin x$	M1	For substituting PI into DE
$\Rightarrow p=\frac{1}{3},\quad q=0$	A1	For correct p and q (which may be implied)
$\Rightarrow y = A\cos 2x + B\sin 2x + \frac{1}{3}\sin x$	B1 √ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
(ii) $(0,0)\Rightarrow A=0$	B1 √	For correct equation in A and/or B f.t. from their GS
$\frac{\mathrm{d}y}{\mathrm{d}x} = 2B\cos 2x + \frac{1}{3}\cos x \Rightarrow \frac{4}{3} = 2B + \frac{1}{3}$	M1	For differentiating their GS and substituting values for x and $\frac{\mathrm{d}y}{\mathrm{d}x}$
$A=0,\ B=\frac{1}{2}$	A1	For correct A and B Allow $A=-\frac{1}{4}\mathrm{i},\ B=\frac{1}{4}\mathrm{i}$ from CF $Ae^{2\mathrm{i}x}+Be^{-2\mathrm{i}x}$
$\Rightarrow y=\frac{1}{2}\sin 2x+\frac{1}{3}\sin x$	A1 **4**	For stating correct solution **CAO**
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2.

5			$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{3}{x}y=x+1$	B1	Divide both sides by x	
			$I=\exp\left(\int \frac{3}{x}\mathrm{d}x\right)=e^{3\ln x}$	M1		
			$=x^3$	A1		A0 means no further marks can be gained
			$x^3\frac{\mathrm{d}y}{\mathrm{d}x}+3x^2y=x^4+x^3$			
			$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3y\right)=\cdots$	M1	Multiply and recognise derivative	
			$\cdots=x^4+x^3$			
				M1	Integrate both sides (their two term polynomial)	condone absent A at this stage
			$x^3y=\frac{1}{5}x^5+\frac{1}{4}x^4+A$	A1		
			$x=1, y=1\Rightarrow A=\frac{11}{20}$	M1	Use condition	
			$y=\frac{1}{5}x^2+\frac{1}{4}x+\frac{11}{20}x^{-3}$	A1 **[8]**		

3.

(i)		$\left(2m^2+3m-2=0\right)\Rightarrow m=\frac{1}{2},-2$	M1	For attempt to solve correct auxiliary equation
		CF = $Ae^{\frac{1}{2}x}+Be^{-2x}$	A1 [2]	For correct CF
(ii)		$\frac{\mathrm{d}y}{\mathrm{d}x}=pe^{-2x}-2pxe^{-2x}$	M1	For differentiating PI twice, using product rule
		$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4pe^{-2x}+4pxe^{-2x}$	A1	For correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$
		$\Rightarrow (-8p+3p+8px-6px-2px)e^{-2x}=5e^{-2x}$	M1	For substituting into DE
		$\Rightarrow p=-1$	A1 [4]	For correct p
(iii)		GS $(y=)Ae^{\frac{1}{2}x}+Be^{-2x}-xe^{-2x}$	B1 FT	For GS soi. FT from CF (2 constants) and p
		$(0,0)\Rightarrow A+B=0$	B1 FT	For correct equation. FT from GS of form $Ae^{\alpha x}+Be^{\beta x}-Cxe^{-2x}$
		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}Ae^{\frac{1}{2}x}-2Be^{-2x}-e^{-2x}+2xe^{-2x}$		
		$\left(0,\frac{\mathrm{d}y}{\mathrm{d}x}=4\right)\Rightarrow\frac{1}{2}A-2B=5$	M1	For differentiating GS and substituting values, using GS of form $Ae^{\alpha x}+Be^{\beta x}-Cxe^{-2x}$
		$\Rightarrow A=2, B=-2$	M1	For solving for A and B(can be gained from incorrect GS)
		$\Rightarrow\ y=2e^{\frac{1}{2}x}-2e^{-2x}-xe^{-2x}$	A1 [5]	For correct solution, including $\boxed{y=}$

4.

(i) Integrating factor $e^{\int \tan x\,(dx)}$	B1	For correct IF
$= e^{-\ln\cos x}$	M1	For integrating to ln form
$= (\cos x)^{-1}$ *OR* $\sec x$	A1	For correct simplified IF **AEF**
$\Rightarrow \frac{d}{dx}\left(y(\cos x)^{-1}\right) = \cos^2 x$	B1√	For $\frac{d}{dx}(y.\text{their IF}) = \cos^3 x$. their IF
$y(\cos x)^{-1} = \int \frac{1}{2}(1+\cos 2x)\,(dx)$	M1 M1	For integrating LHS For attempting to use $\cos 2x$ formula *OR* parts for $\int \cos^2 x\,dx$
$y(\cos x)^{-1} = \frac{1}{2}x + \frac{1}{4}\sin 2x\ (+c)$	A1	For correct integration both sides **AEF**
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c\right)\cos x$	A1 **8**	For correct general solution **AEF**
(ii) $2 = \left(\frac{1}{2}\pi + c\right).-1 \Rightarrow c = -2 - \frac{1}{2}\pi$	M1	For substituting $(\pi, 2)$ into their GS and solve for c
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x - 2 - \frac{1}{2}\pi\right)\cos x$	A1 **2**	For correct solution **AEF**
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5.

(i)		PI: $y = ax\cos 2x + bx\sin 2x$		
		$\frac{dy}{dx} = a\cos 2x - 2ax\sin 2x + b\sin 2x + 2bx\cos 2x$	B1	For correct $\frac{dy}{dx}$ or better
		$\frac{d^2y}{dx^2} = -4a\sin 2x - 4ax\cos 2x + 4b\cos 2x - 4bx\sin 2x$		
		in DE: $-4a\sin 2x - 4ax\cos 2x + 4b\cos 2x - 4bx\sin 2x$ $+4\left(ax\cos 2x + bx\sin 2x\right)$	M1	Differentiate twice and substitute
		compare coefficients: $-4a = 1,\ 4b = 0$	M1	
		$\Rightarrow a = -\frac{1}{4}, b = 0$	A1	
		AE: $\lambda^2 + 4 = 0$ $\lambda = \pm 2i$	M1	For correct auxiliary equation and attempt to solve
		CF: $A\cos 2x + B\sin 2x$	A1	oe form
		GS: $y = \left(A - \frac{1}{4}x\right)\cos 2x + B\sin 2x$	A1ft [7]	Must be real and contain 2 unknowns
(ii)		oscillations	B1	oe (accept sketch) dep consistent with 6(i)
		unbounded	B1 [2]	oe (accept sketch) dep consistent with 6(i) If zero, then sc1 for recognition that xcos2x term becomes dominant
(iii)		If $k \neq 2$ then PI $y = \alpha\cos kx + \beta\sin kx$	B1	
		So bounded oscillations	B1 [2]	oe (accept sketch)