QuestionSchemeMarksA5(a) $(t+4)\frac{dv}{dt}+5v=10(t+4)\Rightarrow\frac{dv}{dt}+\frac{5v}{(t+4)}=10$ M11IF = $e^{\int_{t+4}^{t+4}u} = (t+4)^5 \Rightarrow v(t+4)^5 = \int 10(t+4)^5 dt$ M13 $v(t+4)^5 = \frac{5}{3}(t+4)^6 + c$ A11 $t=0, v=0 \Rightarrow c=-\frac{20480}{3}$ M13 $t=3 \Rightarrow v=\frac{5}{3} \times 7-\frac{20480}{3 \times 7^5}$ M13 $v=11.3(ms^{-1})$ A11(c)For large values of t, the velocity increasesB1(1)E.g.(1)(c)The raindrop may hit an obstacle as it falls • The raindrop is unlikely to be at rest initially • The raindrop mill eventually hit the groundB13M13(a)M1(1)M1U(1)M1M1M1M1M1M1M1M1M1M1M2M1M3M1M3M1M4M1M4M1M4M1M4M1M4M1M4M1M4M1M4M1M4M1M5M4M4M2M4M4M5M4M5M4M5M4M5M4M4M5M4M4M5M4M5M4M5M4M5M4M5M4M5M4M5M4M5M4 <th>1</th> <th></th> <th></th> <th></th>	1			
n(t+4) $\frac{dv}{dt}$ + 5v = 10(t+4) $\Rightarrow \frac{dv}{dt}$ + $\frac{5v}{(t+4)}$ = 10M11S(a) $(t+4)^{\frac{dv}{dt}}$ + 5v = 10(t+4) $\Rightarrow v(t+4)^{\frac{s}{2}} = \int 10(t+4)^{\frac{s}{2}} dt$ M13 $v(t+4)^{\frac{s}{2}} = \frac{5}{3}(t+4)^{\frac{6}{2}} + c$ A11 $t=0, v=0 \Rightarrow c=-\frac{20480}{3}$ M13 $t=3 \Rightarrow v=\frac{5}{3} \times 7-\frac{20480}{3 \times 7^{\frac{s}{2}}}$ M13 $v=11.3$ (ms ⁻¹)A11(c)For large values of t, the velocity increasesB11(c)E.g.(1)(c)The raindrop may hit an obstacle as it falls \bullet B13 \bullet The raindrop is unlikely to be at rest initially \bullet B13(a)The raindrop will eventually hit the ground(1)Notes(a)M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution(1) the rowspan="2">Mits use the solution to the problem to find the velocity after 3 seconds A1: Correct value (b)B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does	Questio	Scheme	Marks	AOs
5(a) $(t+4)\frac{dv}{dt} + 5v = 10(t+4) \Rightarrow \frac{dv}{dt} + \frac{5v}{(t+4)} = 10$ M1 1 IF = $e^{\int \frac{5}{t+4}dt} = (t+4)^5 \Rightarrow v(t+4)^5 = \int 10(t+4)^5 dt$ M1 3 $v(t+4)^5 = \frac{5}{3}(t+4)^6 + c$ A1 1 $t = 0, v = 0 \Rightarrow c = -\frac{20480}{3}$ M1 3 $t = 3 \Rightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^5}$ M1 3 $v = 11.3(ms^{-1})$ A1 1 (c) E.g. (1) (c) E.g. (2) (c) E.g. (2) (c) E.g.	n			
IF = $e^{\int \frac{1}{t+4} dt} = (t+4)^5 \Rightarrow v(t+4)^5 = \int 10(t+4)^5 dt$ M13 $v(t+4)^5 = \frac{5}{3}(t+4)^6 + c$ A11 $t=0, v=0 \Rightarrow c = -\frac{20480}{3}$ M13 $t=3 \Rightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^5}$ M13 $v=11.3(ms^{-1})$ A11(c)For large values of t, the velocity increasesB1(1)(6)(c)E.g.(c)The raindrop may hit an obstacle as it falls• The raindrop may be affected by the wind as it falls• The raindrop will eventually hit the ground(1)(2)(3)(4)(5)(7)(7)(8)(8)(9)(10)(11)(12)(13)(14)(15)(15)(15)(16)(17)(18)(19)(19)(10)(10)(11)(12)(13)(14)(15)(15)(15)(16)(17)(18)(18)(19)(19)(11)(11)(12)(13)(14)(15)(16)(17)(18)(19)(19)(11)(11)(11)(12)(13)(14)(15)(15)(16)(17)(5(a)	$(t+4)\frac{\mathrm{d}v}{\mathrm{d}t} + 5v = 10(t+4) \Longrightarrow \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{5v}{(t+4)} = 10$	M1	1.1b
$v(t+4)^{5} = \frac{5}{3}(t+4)^{6} + c$ A11 $t = 0, v = 0 \Rightarrow c = -\frac{20480}{3}$ M13 $t = 3 \Rightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^{5}}$ M13 $v = 11.3 (ms^{-1})$ A11(6)(6)(b)For large values of t, the velocity increasesB1(c)E.g.(1)• The raindrop may bit an obstacle as it falls• The raindrop may be affected by the wind as it falls• The raindrop will eventually hit the ground(1)(a)(8 ma)M1: Divides through by $(t+4)$ (1)M1: Uses the model to find the integrating factor and attempts the solution of the differential equationA1: Correct solution(find the integrating factor and attempts the solution of the differential equationA1: Correct value(b)B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does		IF = $e^{\int \frac{5}{t+4} dt} = (t+4)^5 \Longrightarrow v(t+4)^5 = \int 10(t+4)^5 dt$	M1	3.1b
$t = 0, v = 0 \Rightarrow c = -\frac{20480}{3}$ M13 $t = 3 \Rightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^3}$ M13 $v = 11.3 (ms^{-1})$ A11 $v = 11.3 (ms^{-1})$ A11(b)For large values of t, the velocity increasesB1(c)E.g.(1)(c)The raindrop may bit an obstacle as it falls• The raindrop may be affected by the wind as it falls• The raindrop will eventually hit the ground(1)(2)(3)• The raindrop will eventually hit the ground(4)• The raindrop will eventually hit the ground(5)• The raindrop will eventually hit the ground• The raindrop will eventually hit the ground <td></td> <td>$v(t+4)^5 = \frac{5}{3}(t+4)^6 + c$</td> <td>A1</td> <td>1.1b</td>		$v(t+4)^5 = \frac{5}{3}(t+4)^6 + c$	A1	1.1b
$t = 3 \Rightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^5}$ M13 $v = 11.3 (\text{ ms}^{-1})$ A11(b)For large values of t, the velocity increasesB11(c)E.g. (1)(1)813• The raindrop may hit an obstacle as it falls • The raindrop may be affected by the wind as it falls • The raindrop will eventually hit the ground(1)(a)Notes(1)(a)Notes(2)(a)Notes the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the velocity after 3 seconds A1: Correct value (b)B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does		$t = 0, \ v = 0 \Longrightarrow c = -\frac{20480}{3}$	M1	3.4
$v = 11.3 (ms^{-1})$ A1 1 (b) For large values of t, the velocity increases B1 1 (c) E.g. (1) (1) (c) The raindrop may hit an obstacle as it falls 81 3 • The raindrop may be affected by the wind as it falls 81 3 • The raindrop will eventually hit the ground (1) (1) (c) E.g. (1) (1) 3 • The raindrop may be affected by the wind as it falls 81 3 • The raindrop will eventually hit the ground (1) (1) (a) (1) (2) (3) M1: Divides through by $(t + 4)$ (1) (8 ma (a) Notes (3) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does		$t = 3 \Longrightarrow v = \frac{5}{3} \times 7 - \frac{20480}{3 \times 7^5}$	M1	3.4
(b) For large values of t , the velocity increases B1 1 (c) E.g. (1) (c) The raindrop may hit an obstacle as it falls B1 3 • The raindrop may be affected by the wind as it falls B1 3 • The raindrop may be affected by the wind as it falls Image: Comparison of the text of tex		$v = 11.3 (ms^{-1})$	A1	1.1b
(b) For large values of t, the velocity increases B1 1 (c) E.g. (1) (1) (c) E.g. 1 1 (c) The raindrop may hit an obstacle as it falls B1 3 • The raindrop may be affected by the wind as it falls B1 3 • The raindrop may be affected by the wind as it falls 1 • The raindrop will eventually hit the ground (1) (a) Notes (1) (a) Notes (8 ma) (a) Notes (8 ma) (b) Solution (1) (2) (b) B1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does Distribution of the raindrop e.g. as t increases so does			(6)	
(c) E.g. (1) • The raindrop may hit an obstacle as it falls • The raindrop is unlikely to be at rest initially B1 3 • The raindrop may be affected by the wind as it falls • The raindrop will eventually hit the ground (1) 3 • The raindrop will eventually hit the ground (1) (1) (1) 3 • The raindrop will eventually hit the ground (1) (1) (1) 3 • The raindrop will eventually hit the ground (1) (1) (1) 3 • The raindrop will eventually hit the ground (1) (1) (1) (1) • The raindrop will eventually hit the ground (1) (1) (1) (1) (1) • The raindrop will eventually hit the ground (1) (1) (1) (1) (1) • The raindrop will eventually hit the ground (1) (1) (1) (1) (1) (1) • (1) • (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1)<	(b)	For large values of <i>t</i> , the velocity increases	B1	1.1b
(c)E.g. • The raindrop may hit an obstacle as it falls • The raindrop is unlikely to be at rest initially • The raindrop may be affected by the wind as it falls • The raindrop will eventually hit the groundB13• The raindrop may be affected by the wind as it falls • The raindrop will eventually hit the ground(1)3• The raindrop will eventually hit the ground(1)(1)• The raindrop will eventually hit the ground(1)• (a)(8 ma)M1: Divides through by $(t + 4)$ (1)M1: Uses the model to find the integrating factor and attempts the solution of the differential equationA1: Correct solution(1)M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b)B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does			(1)	
(1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (2) (2) (3) M1: Divides through by (t + 4) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does	(c)	 E.g. The raindrop may hit an obstacle as it falls The raindrop is unlikely to be at rest initially The raindrop may be affected by the wind as it falls The raindrop will eventually hit the ground 	B1	3.5b
(8 ma Notes (a) M1: Divides through by (t + 4) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does			(1)	
Notes (a) M1: Divides through by (t + 4) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does			(8	marks)
 (a) M1: Divides through by (t + 4) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the velocity after 3 seconds A1: Correct value (b) B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does 		Notes		
	M1: Divic M1: Uses equation A1: Corre M1: Intern M1: Uses A1: Corre (b) B1: Make (c)	les through by $(t + 4)$ the model to find the integrating factor and attempts the solution of the ct solution prets the initial conditions to find the constant of integration their solution to the problem to find the velocity after 3 seconds ct value s a sensible comment regarding the motion of the raindrop e.g. as <i>t</i> incr	different reases so o	ial does v

Topic Z3 Differential equations (Pre-TT A) [51] MARKSCHEME

2.			
(a)	$r = 10 \frac{df}{dt} - 2f \Longrightarrow \frac{dr}{dt} = 10 \frac{d^2f}{dt^2} - 2 \frac{df}{dt}$	M1	2.1
	$10\frac{d^2f}{dt^2} - 2\frac{df}{dt} = -0.2f + 0.4\left(10\frac{df}{dt} - 2f\right)$	M1	2.1
	$\frac{d^2 f}{dt^2} - 0.6 \frac{df}{dt} + 0.1 f = 0^*$	A1*	1.1b
		(3)	
<u>b)</u>	$m^2 - 0.6m + 0.1 = 0 \Rightarrow m = \frac{0.6 \pm \sqrt{0.6^2 - 4 \times 0.1}}{2}$	M1	3.4
	$m = 0.3 \pm 0.1$ i	A1	1.1b
	$f = e^{\alpha t} \left(A \cos \beta t + B \sin \beta t \right)$	M1	3.4
	$f = e^{0.3t} \left(A \cos 0.1t + B \sin 0.1t \right)$	A1	1.1b
		(4)	
[c)	$\frac{df}{dt} = 0.3e^{0.3t} \left(A\cos 0.1t + B\sin 0.1t \right) + 0.1e^{0.3t} \left(B\cos 0.1t - A\sin 0.1t \right)$	M1	3.4
	$r = 10\frac{\mathrm{d}f}{\mathrm{d}t} - 2f$	M1	3.4
	$= e^{0.3t} \left((3A+B)\cos 0.1t + (3B-A)\sin 0.1t \right) - 2e^{0.3t} \left(A\cos 0.1t + B\sin 0.1t \right)$		
	$r = e^{0.3t} \left((A+B)\cos 0.1t + (B-A)\sin 0.1t \right)$	A1	1.1b
		(3)	
(d)(i)	$t=0,f=6 \Longrightarrow A=6$	M1	3.1b
	$t = 0, r = 20 \Longrightarrow B = 14$	M1	3.3
	$r = e^{0.3t} \left(20\cos 0.1t + 8\sin 0.1t \right) = 0$	M1	3.1b
	$\tan 0.1t = -2.5$	A1	1.1b
	2019	A1	3.2a
d)(ii)	3750 foxes	B1	3.4
d)(iii)	e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible	B1	3.5a
		(7)	
		(17 r	narks)

3.			
Question	Scheme	Marks	AOs
8(a)(i)	Container contains 3+0.25t -0.125t = 3 + 0.125t litres after t minutes	B1	3.3
(ii)	Rate of contaminant out = $0.125 \times \frac{m}{3 + 0.125t}$ mg per minute	M1	3.3
	Rate of contaminant in = $0.25 \times (5 - e^{-0.1t})$ mg per minute	B1	2.2a
	$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{5 - \mathrm{e}^{-0.1t}}{4} - \frac{m}{24 + t} *$	A1*	1.1b
		(4)	
(b)	Rearranges to form $\frac{dm}{dt} + \frac{m}{24+t} = \frac{5-e^{-0.1t}}{4}$ and attempts integrating factor (may be by recognition).	M1	3.1a
	I.F. = $\left(e^{\int \frac{1}{24+t} dt} = e^{\ln(24+t)}\right) = 24 + t$	A1	1.1b
	$(24+t)m = \frac{1}{4}\int (24+t)(5-e^{-0.1t}) dt = \frac{1}{4}\int 120+5t-24e^{-0.1t}-te^{-0.1t} dt =$	M1	3.1a
	$=\frac{1}{4}\left(120t+\frac{5t^2}{2}-\frac{24e^{-0.1t}}{-0.1}+\ldots\right)$	A1	1.1b
	$\int t e^{-0.1t} dt = t \frac{e^{-0.1t}}{-0.1} - \int 1 \times \frac{e^{-0.1t}}{-0.1} dt = t \frac{e^{-0.1t}}{-0.1} - \frac{e^{-0.1t}}{(-0.1)^2}$	M1 A1	1.1b 1.1b
	So $(24+t)m = \frac{5}{8}t^2 + 30t + 85e^{-0.1t} + \frac{5}{2}te^{-0.1t} + c$		
	When $t = 0$, $m = 0$ as initially no contaminant in the container, so $0 = 0 + 0 + 85 + 0 + c \Rightarrow c = -85$	M1	3.4
	$m = \frac{1}{24+t} \left(\frac{5}{8}t^2 + 30t + 85e^{-0.1t} + \frac{5}{2}te^{-0.1t} - 85 \right)$	A1	2.2b
		(8)	
(c)	When $t = 30 m = 25.65677$ and $V = 6.75$, hence the concentration is 3.80 mg per litre.	M1	3.4
	This resembles the measured value very closely and could easily be explained by minor inaccuracies in measurements, so the model seems to be suitable over this timeframe.	A1	3.5a
		(2)	
	(14 mar		

4.			
(a)(i)	Weight = mass × g \Rightarrow $m = \frac{30000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	M1	3.3
(ii)	$\frac{dx}{dt} = 40\cos t + 20\sin t, \ \frac{d^2x}{dt^2} = -40\sin t + 20\cos t$	M1	1.1b
	$3(-40\sin t + 20\cos t) + 4(40\cos t + 20\sin t) + 40\sin t - 20\cos t =$	M1	1.1b
	$= 200 \cos t$ so PI is $x = 40 \sin t - 20 \cos t$	A1*	2.1
	or		
	Let $x = a\cos t + b\sin t$ $\frac{dx}{dt} = -a\sin t + b\cos t, \frac{d^2x}{dt^2} = -a\cos t - b\sin t$	М1	1.1b
	$4b - 2a = 200, -2b - 4a = 0 \Longrightarrow a = \dots, b = \dots$	M1	2.1
	$x = 40\sin t - 20\cos t$	A1*	1.1b
(iii)	$3\lambda^2 + 4\lambda + 1 = 0 \Longrightarrow \lambda = -1, -\frac{1}{3}$	M1	1.1b
	$x = A\mathrm{e}^{-t} + B\mathrm{e}^{-\frac{1}{3}t}$	A1	1.1b
	x = PI + CF	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
		(8)	
(b)	$t = 0, x = 0 \Longrightarrow A + B = 20$	M1	3.4
	$x = 0, \frac{dx}{dt} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40\cos t + 20\sin t = 0$ $\implies A + \frac{1}{3}B = 40$	М1	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
	$t = 9 \Rightarrow x = 33m$	A1	3.4
		(4)	
		(12 u	narks)