Topic Z4 Complex numbers (Pre-TT A) [50]

1.

(i) Write down, in cartesian form, the roots of the equation $z^4 = 16$. **[2]**

(ii) Hence solve the equation $w^4 = 16(1 - w)^4$, giving your answers in cartesian form. **[5]**

(Total 7 marks)

2.

(i) Use de Moivre's theorem to prove that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ [4]

(ii) Hence find the largest positive root of the equation

$$64x^6 - 96x^4 + 36x^2 - 3 = 0,$$

giving your answer in trigonometrical form. [4]

(Total 8 marks)

3.

(i) By expressing $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, prove that

$$\sin^3\theta\cos^2\theta \equiv -\tfrac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta).$$ **[6]**

(ii) Hence show that all the roots of the equation

$$\sin 5\theta = \sin 3\theta + 2\sin\theta$$

are of the form $\theta = \dfrac{n\pi}{k}$, where n is any integer and k is to be determined. **[3]**

(Total 9 marks)

4.

(i) Solve the equation $z^4 = 64(\cos\pi + i\sin\pi)$, giving your answers in polar form. [2]

(ii) By writing your answers to part **(i)** in the form $x + iy$, find the four linear factors of $z^4 + 64$. [4]

(iii) Hence, or otherwise, express $z^4 + 64$ as the product of two real quadratic factors. [3]

(Total 9 marks)

5.

The integrals C and S are defined by

$$C = \int_0^{\frac{1}{2}\pi} e^{2x} \cos 3x \, dx \qquad \text{and} \qquad S = \int_0^{\frac{1}{2}\pi} e^{2x} \sin 3x \, dx.$$

By considering $C + iS$ as a single integral, show that

$$C = -\tfrac{1}{13}(2 + 3e^{\pi}),$$

and obtain a similar expression for S. [8]

(You may assume that the standard result for $\int e^{kx} \, dx$ remains true when k is a complex constant, so that $\int e^{(a+ib)x} \, dx = \dfrac{1}{a+ib} e^{(a+ib)x}$.)

(Total 8 marks)

6.

In an Argand diagram, the complex numbers $0, z$ and $ze^{\frac{1}{6}i\pi}$ are represented by the points O, A and B respectively.

(i) Sketch a possible Argand diagram showing the triangle OAB. Show that the triangle is isosceles and state the size of angle AOB. **[4]**

The complex numbers $1 + i$ and $5 + 2i$ are represented by the points C and D respectively. The complex number w is represented by the point E, such that $CD = CE$ and angle $DCE = \frac{1}{6}\pi$.

(ii) Calculate the possible values of w, giving your answers exactly in the form $a + bi$. **[5]**

(Total 9 marks)