Topic Z5 polar coordinates and series (Post-TT A) [53]

1.

Find $\sum_{r=1}^{n}(4r^3+6r^2+2r)$, expressing your answer in a fully factorised form. **[6]**

(Total 6 marks)

2.

Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. **[5]**

(Total 5 marks)

3.

(i) Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

(iii) State, giving a brief reason, whether the series $\sum_{r=1}^{\infty} \dfrac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

(Total 9 marks)

4.

(i) Show that $\dfrac{2}{r} - \dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$ [6]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \dfrac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\sum_{r=N+1}^{\infty} \dfrac{3r+4}{r(r+1)(r+2)} = \dfrac{7}{10}$, find the value of N. [4]

(Total 9 marks)

5.

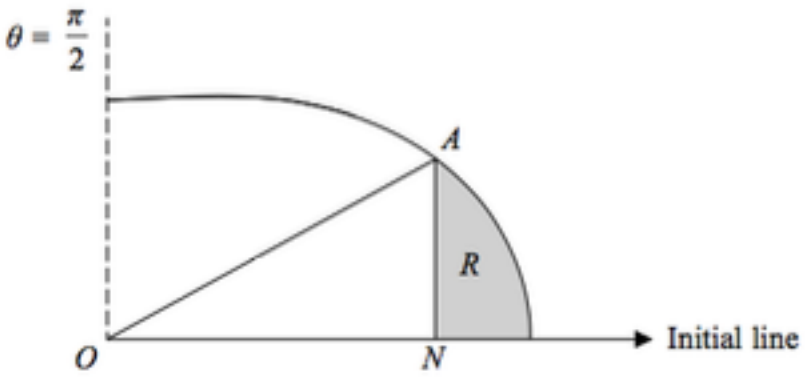

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(9)

(Total 13 marks)

6.

The equation of a curve, in polar coordinates, is

$$r = \sqrt{3} + \tan\theta, \quad \text{for } -\tfrac{1}{3}\pi \leqslant \theta \leqslant \tfrac{1}{4}\pi.$$

(i) Find the equation of the tangent at the pole. [2]

(ii) State the greatest value of r and the corresponding value of θ. [2]

(iii) Sketch the curve. [2]

(iv) Find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$. [5]

(Total 11 marks)