# Topic Z5 polar coordinates and series (Post-TT A) [53] MARKSCHEME

1.

| | | | |
|---|---|---|---|
| $n^2(n+1)^2 + n(n+1)(2n+1) + n(n+1)$ | M1 | | Express as sum of 3 terms |
| | A1 | | 2 correct unsimplified terms |
| | A1 | | 3rd correct unsimplified term |
| $n(n+1)^2(n+2)$ | M1 | | Attempt to factorise |
| | A1ft | | Two factors found, ft their quartic |
| | A1 | 6 | Correct final answer a.e.f. |
| | | **6** | |

2.

| | | | |
|---|---|---|---|
| | B1 | | Establish result true for $n = 1$ or $n = 2$ |
| | M1 | | Add next term to given sum formula |
| | M1 | | Attempt to factorise or expand and simplify to correct expression |
| | A1 | | Correct expression obtained |
| | A1 | **5** | Specific statement of induction conclusion |
| | | **5** | |

3.

**(i)**

| | | | |
|---|---|---|---|
| | M1 | | Attempt to rationalise denominator or cross multiply |
| | A1 | **2** | Obtain **given** answer correctly |

**(ii)**

$\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1)$

| | | | |
|---|---|---|---|
| | M1 | | Express terms as differences using (i) |
| | M1 | | Attempt this for at least 1st three terms |
| | A1 | | 1st three terms all correct |
| | A1 | | Last two terms all correct |
| | M1 | | Show pairs cancelling |
| | A1 | **6** | Obtain correct answer, in terms of $n$ |

**(iii)**

| | | | |
|---|---|---|---|
| | B1 | **1** | Sensible statement for divergence |
| | | **9** | |

4.

| | | | |
|---|---|---|---|
| (i) | M1 | | Attempt to combine 3 fractions |
| | A1 | 2 | Obtain given answer correctly |
| (ii) | M1 | | Express at least first 3 terms using (i) |
| | A1 | | All terms correct |
| | M1 | | Express at least last 2 terms using (i) |
| | A1 | | All terms correct in terms of $n$ |
| | M1 | | Show that correct terms cancel |
| $2+1-\frac{1}{2}-\frac{2}{n+1}-\frac{1}{n+2}$ | A1 | 6 | Obtain unsimplified correct answer |
| (iii) $\frac{5}{2}$ | B1ft | 1 | Obtain correct answer from their (ii) |
| (iv) $\frac{2}{N+1}+\frac{1}{N+2}=\frac{7}{10}$ | B1ft | | Their (iii) – their (ii) |
| $7N^2-9N-36=0$ | M1 | | Attempt to clear fractions & solve equation, Obtain correct simplified equation |
| $N=3$ | A1 | | Obtain only the correct answer |
| | A1 | 4 | |
| | | **13** | |

5.

| | | |
|---|---|---|
| $4+\cos 2\theta=\frac{9}{2}\Rightarrow\theta=...$ | M1 | 3.1a |
| $\theta=\frac{\pi}{6}$ | A1 | 1.1b |
| $\frac{1}{2}\int(4+\cos 2\theta)^2\mathrm{d}\theta=\frac{1}{2}\int(16+8\cos 2\theta+\cos^2 2\theta)\mathrm{d}\theta$ | M1 | 3.1a |
| $\cos^2 2\theta=\frac{1}{2}+\frac{1}{2}\cos 4\theta\Rightarrow A=\frac{1}{2}\int\left(16+8\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)\mathrm{d}\theta$ | M1 | 3.1a |
| $=\frac{1}{2}\left[16\theta+4\sin 2\theta+\frac{\sin 4\theta}{8}+\frac{\theta}{2}\right]$ | A1 | 1.1b |
| Using limits 0 and their $\frac{\pi}{6}$: $\frac{1}{2}\left[\frac{33\pi}{12}+2\sqrt{3}+\frac{\sqrt{3}}{16}-(0)\right]$ | M1 | 1.1b |
| Area of triangle $=\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{81}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}$ | M1 | 3.1a |
| Area of $R=\frac{33\pi}{24}+\frac{33\sqrt{3}}{32}-\frac{81\sqrt{3}}{32}$ | M1 | 1.1b |
| $=\frac{11}{8}\pi-\frac{3\sqrt{3}}{2}\left(p=\frac{11}{8},\ q=-\frac{3}{2}\right)$ | A1 | 1.1b |
| | | **(9 marks)** |

6.

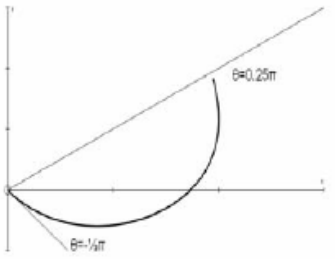
(i) Attempt to solve $r$=0, tan $\theta$ = - √3
Get $\theta$ = - $^1/_3$ π only

M1 Allow ±√3
A1 Allow – 60°

(ii) $r$ = √3 + 1 when $\theta$ = ¼π

B1,B1 AEF for $r$ , 45° for $\theta$

(iii)




B1 Correct $r$ at correct end-values of $\theta$;
Ignore extra $\theta$ used

B1 Correct shape with $r$ not decreasing

(iv) Formula with correct $r$ used
Replace $\tan^2\theta = \sec^2\theta - 1$
Attempt to integrate their expression

Get $\theta$ + √3 ln sec$\theta$ + ½ tan$\theta$
Correct limits to ¼π + √3 ln√2 + ½

M1 $r^2$ may be implied
B1
M1 Must be 3 different terms leading to any 2 of $a\theta$ + $b$ ln (sec$\theta$/cos$\theta$) + $c$ tan $\theta$
A1 Condone answer x2 if ½ seen elsewhere
A1 cao; AEF