Topic Z5 polar coordinates and series (Post-TT B) [53]

1.

Find $\sum_{r=1}^{n} (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. [7]

(Total 7 marks)

2.

(i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$ [2]

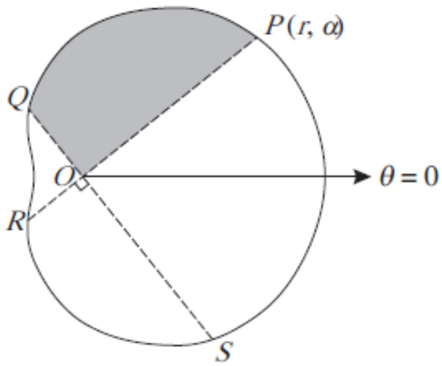
(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}.$$ [4]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

(Total 7 marks)

3.



The diagram shows the curve with equation, in polar coordinates,

$$r = 3 + 2\cos\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

The points P, Q, R and S on the curve are such that the straight lines POR and QOS are perpendicular, where O is the pole. The point P has polar coordinates (r, α).

(i) Show that $OP + OQ + OR + OS = k$, where k is a constant to be found. [3]

(ii) Given that $\alpha = \frac{1}{4}\pi$, find the exact area bounded by the curve and the lines OP and OQ (shaded in the diagram). [5]

(Total 8 marks)

4.

Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [5]

(Total 5 marks)

5.

Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **[6]**

(Total 4 marks)

6.

(i) Show that $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ **[6]**

(iii) Show that $\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$. **[3]**

(Total 11 marks)

7.

The equation of a curve, in polar coordinates, is

$$r = \sec\theta + \tan\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{3}\pi.$$

(i) Sketch the curve. [2]

(ii) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [6]

(iii) Find a cartesian equation of the curve. [3]

(Total 11 marks)