Topic Z5 polar coordinates and series (Post-TT B) [53] MARKSCHEME

1.

		$\frac{1}{2}n(n+1)(2n+1)-\frac{3}{2}n(n+1)+2n$	M1 M1 A1 A1	Express as sum of 3 series Use standard series results, at least 1 correct Two terms correct Third term correct
		$n(n^2+1)$	M1 A2 [7]	Obtain factor of n Obtain correct answer c.a.o. Allow A1 for $\frac{1}{2(2n^2+2)}$

2.

(i)	$\frac{(r+1)^2-r(r+2)}{(r+2)(r+1)}$ $\frac{1}{(r+1)(r+2)}$	M1 A1	 2	Show correct process for subtracting fractions Obtain given answer correctly
(ii)	*EITHER* $\frac{2}{3}-\frac{1}{2}+\frac{3}{4}-\frac{2}{3}\ \ldots\ \frac{n+1}{n+2}-\frac{n}{n+1}$ $\frac{n+1}{n+2}-\frac{1}{2}$	M1 A1 M1 A1	 4	Express terms as differences using (i) At least first two and last term correct Show or imply that pairs of terms cancel Obtain correct answer in any form
	OR	M2 A1A1		State that $\sum_{r=1}^{n} u_r = f(n+1)-f(1)$ Each term correct
(iii)	$\frac{1}{2}$	B1 ft	1 **7**	Obtain value from their sum to n terms

3.

(i)	$OP = 3 + 2\cos\alpha$ $OQ = 3 + 2\cos(\frac{1}{2}\pi+\alpha)$ $= 3 - 2\sin\alpha$	M1	Any other unsimplified value
	Similarly $OR = 3 - 2\cos\alpha$ $OS = 3 + 2\sin\alpha$	M1	Attempt at simplification of at least two correct expressions
	Sum = 12	A1	cao
(ii)	Correct formula with attempt at r^2	M1	Need not be expanded, but three terms if it is
	Square r correctly	A1	
	Attempt to replace $\cos^2\theta$ with $a(\cos 2\theta \pm 1)$	M1	
	Integrate their expression	A1√	Need three terms
	Get $11\pi/4 - 1$	A1	cao

4.

	B1		Establish result true for n = 1 or 2
	M1*		Add next term to given sum formula
	DM1		Combine with correct denominator
	A1		Obtain correct expression convincingly
	A1	5	Specific statement of induction conclusion, provided 1st 4 marks earned
	5		

5.

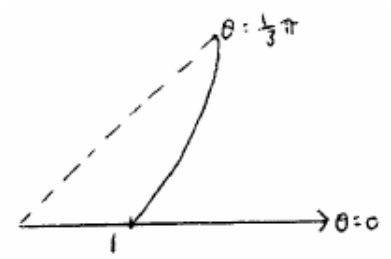
	M1		Express as sum of three series
	M1		Use standard results
$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	A1		Obtain correct unsimplified answer
	M1		Attempt to factorise
	A1		Obtain at least factor of $n(n+1)$
$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	6	Obtain fully factorised correct answer
	6		

6.

(i)	M1		Use correct denominator
	A1	2	Obtain **given** answer correctly
(ii)	M1		Express terms as differences using (i)
	M1		Do this for at least 3 terms
	A1		First 3 terms all correct
	A1		Last 2 terms all correct
$\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}$	M1		Show relevant cancelling
	A1	6	Obtain correct answer a.e.f.
(iii) $\frac{1}{2}$	B1ft		S_∞ stated or start at n + 1 as in (ii)
$\frac{1}{n+1}-\frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
$\frac{1}{(n+1)(n+2)}$	A1	3	Obtain **given** answer correctly
	11		

7.

(i)



B1 Shape for correct θ; ignore other θ Used; start at $(r,0)$

B1 $\theta=0$, $r=1$ and increasing r

(ii) U se correct formula with correct r — B1

$\int \sec^2 x\, dx = \tan x$ used — B1

Quote $f2 \sec x \tan x\, dx = 2\sec x$ — B1 Or sub. correctly

Replace $\tan^2 x$ by $\sec^2 x - 1$ to integrate — M1

Reasonable attempt to integrate 3 terms And to use limits correctly — M1

Get $\sqrt{3} + 1 - {}^1/_6\pi$ — Al Exact only

(iii) Use $x = r\cos\theta$, $y = r\sin\theta$, $r = (x^2+y^2)^{1/2}$ — M1

Reasonable attempt to eliminate r,θ — M1

Get $y = (x-1)\sqrt{(x^2+y^2)}$ — A1 Or equivalent