

Topic Z5 polar coordinates and series (Pre-TT A) [46]

1.

Prove that

$$\sum_{r=1}^n \frac{1}{(r+1)(r+3)} = \frac{n(an+b)}{12(n+2)(n+3)}$$

where a and b are constants to be found.

(5)

(Total 5 marks)

2.

Find $\sum_{r=1}^n r^2(r-1)$, expressing your answer in a fully factorised form.

[6]

(Total 6 marks)

3.

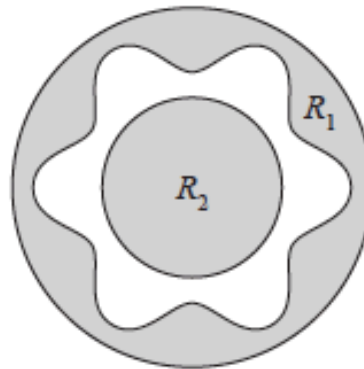


Figure 1

Figure 1 shows the design for a new type of security wheel nut for a car. The inner circle has a radius of 5 mm and the outer circle has a radius of 10 mm. The curve, C , between the two circles, is modelled by the polar equation

$$r = 7.5 + 1.5 \cos 6\theta \quad 0 \leq \theta < 2\pi$$

where r is measured in millimetres.

The regions R_1 and R_2 are shown shaded in Figure 1 and both regions must be coated in a special paint.

The region R_1 is enclosed between the outer circle and C .

The region R_2 is enclosed by the inner circle.

Find the area that must be coated in the special paint, according to the model.

Give your answer in cm^2 to 2 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(7)

(Total 7 marks)

4.

Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n 4 \times 3^r = 6(3^n - 1)$. [5]

(Total 5 marks)

5.

Given that $\sum_{r=1}^n (ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b . [5]

(Total 5 marks)

6.

A company operating a coal mine is concerned about the mine running out of coal. It is estimated that 2.5 million tonnes of coal are left in the mine. The company wishes to mine all of this coal in 20 years.

In order to mine the coal in a regulated manner, the company models the amount of coal to be mined in the coming years by the formula

$$M_r = \frac{10}{r^2 + 8r + 15}$$

where M_r is the amount of coal, in millions of tonnes, mined in year r , with the first year being year 1

(a) Show that, according to the model, the total amount of coal, in millions of tonnes, mined in the first n years is given by

$$T_n = \frac{9n^2 + 41n}{k(n+4)(n+5)}$$

where k is a constant to be determined.

(6)

(b) Explain why, according to this model, the mine will never run out of coal.

(2)

The company decides to mine an extra fixed amount each year so that all the coal will be mined in exactly 20 years.

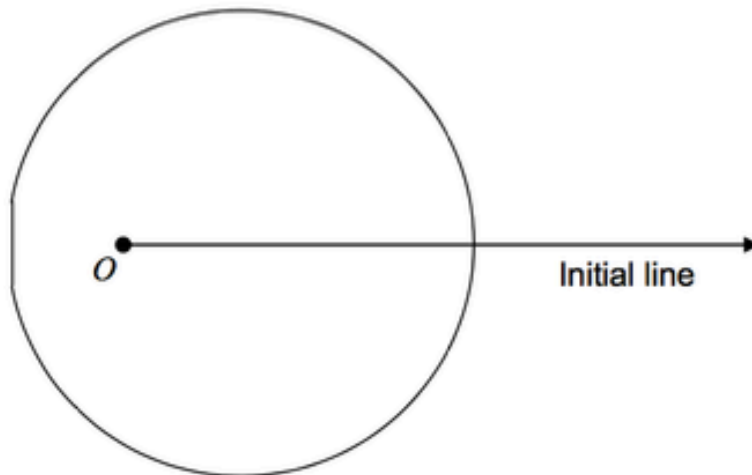
(c) Refine the formula for M_r so that 2.5 million tonnes of coal will be exhausted in exactly 20 years of mining.

(2)

(Total 10 marks)

7.

The diagram shows a sketch of a curve C , the pole O and the initial line.



The polar equation of C is $r = 4 + 2 \cos \theta$, $-\pi \leq \theta \leq \pi$

- (a) Show that the area of the region bounded by the curve C is 18π

[4 marks]

- (b) Points A and B lie on the curve C such that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and AOB is an equilateral triangle.

Find the polar equation of the line segment AB

[4 marks]

(Total 8 marks)