Topic Z5 polar coordinates and series (Pre-TT A) [46] MARKSCHEME

1.

$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = \ldots, B = \ldots$	M1
$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2\times2} - \frac{1}{2\times4} + \frac{1}{2\times3} - \frac{1}{2\times5} + \ldots + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1
$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1
$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1
$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1
	(5)

2.

	M1	Express as difference of two series
	M1	Use standard results
$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1	Correct unsimplified answer
	M1	Attempt to factorise
	A1	At least factor of $n(n+1)$
$\frac{1}{12}n(n+1)(3n+2)(n-1)$	A1	Obtain correct answer
	6	

3.

3	Area enclosed by curve $= \frac{1}{2}\int (7.5+1.5\cos 6\theta)^2 \, d\theta$	M1	3.1a
	$(7.5+1.5\cos 6\theta)^2 = 56.25+22.5\cos 6\theta+2.25\cos^2 6\theta$ $= 56.25+22.5\cos 6\theta+2.25\left(\frac{\cos 12\theta+1}{2}\right)$	M1	2.1
	$\frac{1}{2}\int (7.5+1.5\cos 6\theta)^2 \, d\theta = \frac{459}{16}\theta+\frac{15}{8}\sin 6\theta+\frac{3}{64}\sin 12\theta(+c)$	A1ft	1.1b
	Area enclosed by curve $= \left[\frac{459}{16}\theta+\frac{15}{8}\sin 6\theta+\frac{3}{64}\sin 12\theta\right]_0^{2\pi}$	M1	3.1a
	$= \frac{459\pi}{8} (= 180.24...)$	A1	1.1b
	Total shaded area $= \pi\times 10^2 - "\frac{459\pi}{8}" + \pi\times 5^2$ = 314.15… – 180.24… + 78.53…	M1	3.1a
	$= \frac{541\pi}{8}\,\text{mm}^2 = 2.12\,\text{cm}^2$	A1	3.2a
		(7)	
			(7 marks)

4.

			B1 M1* DepM1 A1 B1 [5]	Verify result true when $n = 1$ Add next term in series Attempt to obtain 3^{k+1} correctly Show sufficient working to justify correct expression Clear statements of Induction processes, but 1st 4 marks must all be earned.

5.

$\frac{a}{6}n(n+1)(2n+1)+bn$ $a = 6 \;\; b = -3$	M1 A1 M1 A1 A1	 5 **5**	Consider sum as two separate parts Correct answer a.e.f. Compare co-efficients Obtain correct answers

6.

2(a)	A correct method to sum the series, most likely by the method of differences. Look for $\frac{10}{r^2+8r+15}=\frac{A}{r+3}+\frac{B}{r+5}\Rightarrow A=..,B=..$ followed by an attempt at the sum (or with 1 instead of 10). (Induction may be attempted – see alt for (a).)	M1	3.1a
	$\frac{10}{r^2+8r+15}=\frac{5}{r+3}-\frac{5}{r+5}$ or $\frac{1}{r^2+8r+15}=\frac{1/2}{r+3}-\frac{1/2}{r+5}$	B1	1.1b
	$\sum_{r=1}^{n}\frac{10}{r^2+8r+15}=5\sum_{r=1}^{n}\left(\frac{1}{r+3}-\frac{1}{r+5}\right)$ $=5\left[\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{6}-\frac{1}{8}\right)+\quad+\left(\frac{1}{n+3}-\frac{1}{n+5}\right)\right]$	M1	2.1
	$=5\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{n+4}-\frac{1}{n+5}\right)$	A1ft	1.1b
	$=5\left(\frac{5(n+4)(n+5)+4(n+4)(n+5)-20(n+5)-20(n+4)}{20(n+4)(n+5)}\right)=\ldots$	M1	2.1
	$=\frac{9n^2+41n}{4(n+4)(n+5)}$ (So k = 4)	A1	1.1b
		(6)	
(b)	As $n\to\infty$, $T_n\to\frac{9}{4}$ or appropriate investigation tried.	M1	3.4
	Since the sum is increasing towards $\frac{9}{4}$ which is strictly less than 2.5 T_n can never reach 2.5, so the 2.5 million remaining tonnes of coal will not all be mined no matter how long the company keeps mining.	A1	3.2b
		(2)	
(c)	In the first 20 years $T_{20}=\frac{221}{120}$ million tonnes of coal have been mined, so $2.5-\frac{221}{120}=\frac{79}{120}$ tonnes remain.	M1	2.2b
	Hence $\frac{79}{120\times 20}$ extra tonnes per year need mining, so the new model is $M_r=\frac{79}{2400}+\frac{10}{r^2+8r+15}$.	A1ft	3.5c
		(2)	
			(10 marks)

7.

(a)	Uses $\frac{1}{2}\int r^2 \, d\theta$ or $\int_0^{\pi} r^2 \, d\theta$ OE	AO1.1a	M1	$\frac{1}{2}\int (4+2\cos\theta)^2 \, d\theta$ $\frac{1}{2}\int_{-\pi}^{\pi}(16+16\cos\theta+4\cos^2\theta) \, d\theta$
	Rewrites $\cos^2\theta$ in terms of $\cos 2\theta$	AO1.1a	M1	$\int_{-\pi}^{\pi}(8+8\cos\theta+(1+\cos 2\theta)) \, d\theta$
	Correctly integrates *their* expression, ft wrong non-zero coefficients.	AO1.1b	A1F	$= \left[8\theta+8\sin\theta+\theta+\frac{1}{2}\sin 2\theta\right]_{-\pi}^{\pi}$
	Obtains required answer from fully correct mathematical argument	AO2.1	R1	$=\left[9\theta+8\sin\theta+\frac{1}{2}\sin 2\theta\right]_{-\pi}^{\pi}$ $=(9\pi+0+0)-(-9\pi+0+0)$ $= 18\pi$
(b)	Selects appropriate method to determine polar equation by equating OA and OB to find θ	AO3.1a	M1	Let $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ $OA = OB \Rightarrow r_1 = r_2$ $\Rightarrow 4+2\cos\theta_1 = 4+2\cos\theta_2$
	Uses the above to find two values of θ and hence deduce the lengths of OA and OB Award this mark for correct deduction using 'their' values of θ	AO2.2a	R1	$\Rightarrow \theta_1 = -\theta_2$ Angle $AOB = \frac{\pi}{3} \Rightarrow \theta_1 - \theta_2 = \frac{\pi}{3}$ $\Rightarrow \theta_1 = \frac{\pi}{6}$ and $\theta_2 = -\frac{\pi}{6}$ $OA = OB = 4+\sqrt{3}$
	Uses the correct polar equation for a perpendicular line $r = d\sec\theta$	AO3.1a	M1	AB is perpendicular to the initial line Polar equation of AB is $r\cos\theta = \left(4+\sqrt{3}\right)\frac{\sqrt{3}}{2}$ for $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$
	Obtains a correct equation for AB (including correct specified range) CAO	AO1.1b	A1	
	Total		**8**	