

Topic Z5 polar coordinates and series (Pre-TT B) [55]

1.

Find $\sum_{r=1}^n (2r-1)^2$, expressing your answer in a fully factorised form. [6]

(Total 6 marks)

2.

(a) Prove that, for all positive integers n ,

$$\sum_{r=1}^n \frac{1}{(5r-2)(5r+3)} \equiv \frac{n}{a(bn+c)}$$

where a , b and c are integers to be determined.

(5)

(b) Hence, showing your working, find the exact value of

$$\sum_{r=10}^{50} \frac{1}{(5r-2)(5r+3)}$$

(2)

(Total 7 marks)

3.

Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n r(3r+1) = n(n+1)^2$. [5]

(Total 5 marks)

4.

The equation of a curve, in polar coordinates, is

$$r = 1 + \cos 2\theta, \quad \text{for } 0 \leq \theta < 2\pi.$$

(i) State the greatest value of r and the corresponding values of θ . [2]

(ii) Find the equations of the tangents at the pole. [2]

(iii) Find the exact area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. [5]

(iv) Find, in simplified form, the cartesian equation of the curve. [4]

(Total 13 marks)

5.

Given that $\sum_{r=1}^n (ar^3 + br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b . [6]

(Total 6 marks)

6.

(i) Show that $(r + 2)! - (r + 1)! = (r + 1)^2 \times r!$. [3]

(ii) Hence find an expression, in terms of n , for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots + (n + 1)^2 \times n!. \quad [4]$$

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots$$

converges.

[1]

(Total 8 marks)

7.

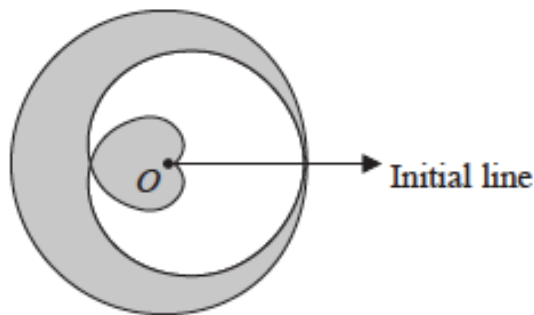


Figure 1

Figure 1 shows a sketch for the design of a logo. The logo is defined by the polar curve with equation

$$r = \sin\left(\frac{\theta}{6}\right) \quad 0 \leq \theta \leq 6\pi$$

The inner closed section and outer closed section of the curve, shown shaded in Figure 1, are to be coloured the same colour. The remaining section is to be left clear.

(a) Use algebraic integration to find the area of the coloured sections of the logo.

(6)

A copy of this logo is to be painted on a white wall of a building such that the total width of the logo is 12 m.

Tins of coloured paint with an advertised minimum coverage area of 30 m^2 are to be used to paint the coloured sections of the logo onto the wall. Given that two coats of paint will be needed,

(b) find the minimum number of tins of this paint that should be bought to ensure that the coloured sections of the logo can be painted onto the wall.

(4)

(Total 10 marks)