Topic Z5 polar coordinates and series (Pre-TT B) [55] MARKSCHEME

1.

Either	M1		Express as a sum of 3 terms
	M1		Use standard sum results
$\frac{2}{3}n(n+1)(2n+1)-2n(n+1)+n$	A1		Correct unsimplified answer
	M1		Attempt to factorise
	A1		Obtain at least factor of n and a quadratic
$\frac{1}{3}n(2n-1)(2n+1)$	A1	6	Obtain correct answer a.e.f.
Or			
$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$	M1		Express as difference of 2 $\sum r^2$ series
	M1		Use standard result
$\frac{1}{6}\times 2n(2n+1)(4n+1)-4\times\frac{1}{6}n(n+1)(2n+1)$	A1		Correct unsimplified answer
	M1		Attempt to factorise
	A1		Obtain at least factor of n
$\frac{1}{3}n(2n-1)(2n+1)$	A1		Obtain correct answer

6

2.

4(a)	$\frac{1}{(5r-2)(5r+3)} \equiv \frac{A}{5r-2}+\frac{B}{5r+3} \Rightarrow A=\ldots,\ B=\ldots,$ $\left(\text{NB } A=\frac{1}{5}\quad B=-\frac{1}{5}\right)$	M1	3.1a
	$\sum_{r=1}^{n}\frac{1}{(5r-2)(5r+3)}$ $\frac{1}{5}\left(\frac{1}{3}-\frac{1}{8}+\frac{1}{8}-\frac{1}{13}+\ldots+\frac{1}{5n-7}-\frac{1}{5n-2}+\frac{1}{5n-2}-\frac{1}{5n+3}\right)$	M1	2.1
	$=\frac{1}{5}\left(\frac{1}{3}-\frac{1}{5n+3}\right)$	A1	1.1b
	$=\frac{1}{5}\left(\frac{5n+3-3}{3(5n+3)}\right)$	M1	1.1b
	$=\frac{n}{3(5n+3)}$	A1	2.2a
		(5)	
(b)	$\sum_{r=10}^{50}\frac{1}{(5r-2)(5r+3)}=\text{f}(50)-\text{f}(9\text{ or }10)$	M1	1.1b
	$=\frac{50}{3(5\times50+3)}-\frac{9}{3(5\times9+3)}=\frac{41}{12144}$	A1	1.1b
		(2)	
			(7 marks)

3.

4				B1	Show sufficient working to verify result true when $n=1$	
			$k(k+1)^2+(k+1)(3k+4)$	M1*	Add next term in series	
				DM1	Attempt to factorise their expression	
			$(k+1)(k+2)^2$	A1	Sufficient working to obtain this correct answer	
				B1	Clear statement of induction process, provided previous 4 marks earned	
				[5]		

4.

8 (i) Max. $r=2$ at $\theta=0$ and π	B1,B1	Two θ needed (rads only); ignore θ out of range
(ii) Solve $r=0$ for θ, giving $\theta=\frac{1}{2}\pi$ and $^3/_2\pi$	M1,A1	Two θ needed (rads only); ignore θ out of range
(iii) Use correct formula with correct r	M1	
Expand r	M1	
Get $\int A + B\cos 2\theta + C\cos 4\theta \, d\theta$	M1	$C \neq 0$
Integrate their expression correctly	M1√	
Get $3\pi/8$	A1	cao
(iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar	M1	
Use $\cos\theta = x/r$ and/or $\sin\theta = y/r$	M1	
Simplify to $(x^2+y^2)^{1.5} = 2x^2$ or similar	M1,A1	

5.

Either	B1	Correct value for $\sum r$ stated or used
	M1	Express as sum of two series
$\frac{a}{4}n^2(n+1)^2+\frac{bn}{2}(n+1)$	A1	Obtain correct unsimplified answer
	M1	Compare coefficients or substitute values for n
$a=4 \quad b=-4$	A1 A1 **6**	Obtain correct answers
Or		
	M1	Use 2 values for n
$a+b=0 \quad 4a+b=12$	A1 A1	Obtain correct equations
	M1	Solve simultaneous equations
$a=4 \quad b=-4$	A1 A1	Obtain correct answers
	6	

6.

(i)	M1		Factor of $r!$ or $(r+1)!$ seen
	A1		Factor of $(r+1)$ found
$(r+1)^2 r!$	A1	3	Obtain given answer correctly
(ii)	M1		Express terms as differences using
	A1		(i)
	M1		At least 1st two and last term correct
$(n+2)! - 2!$	A1	4	Show that pairs of terms cancel
(iii)	B1ft	1	Obtain correct answer in any form
		8	Convincing statement for non-converging, ft their (ii)

7.

3(a)	Correct overall strategy employed, eg. $A=2\times\left(\frac{1}{2}\int_{2\pi}^{3\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta-\frac{1}{2}\int_{4\pi}^{5\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta+\frac{1}{2}\int_{0}^{\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta\right)$	M1	3.1a
	Evidence of use of $\frac{1}{2}\int\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta$	B1	1.1a
	$\int\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta=\int\frac{1}{2}\left(1-\cos\left(\frac{\theta}{3}\right)\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\left(\theta-3\sin\left(\frac{\theta}{3}\right)\right)$	A1	1.1b
	$A=\left(2\times\frac{1}{2}\right)\times\frac{1}{2}\left[\left((3\pi-0)-\left(2\pi-\frac{3\sqrt{3}}{2}\right)\right)-\left(\left(5\pi+\frac{3\sqrt{3}}{2}\right)-\left(4\pi+\frac{3\sqrt{3}}{2}\right)\right)+\left(\left(\pi-\frac{3\sqrt{3}}{2}\right)-(0)\right)\right]$	M1	2.1
	$=\frac{\pi}{2}$	A1	1.1b
		(6)	
(b)	Area of painting on wall = (area curve)×(12/(width of curve))^2 with their area and width.	M1	3.1a
	Width of curve $\left(=\sin\left(\frac{3\pi}{6}\right)+\sin\left(\frac{2\pi}{6}\right)\right)=1+\frac{\sqrt{3}}{2}=1.866...$	B1	1.1b
	So as two coats needed, total area of paint required $=2\times\frac{\pi}{2}\times41.354$ =129.92… m^2	M1	2.2a
	So 5 tins of paint will be needed.	A1	3.2a
		(4)	
			(10 marks)