Write yours and your teacher's name at the top of your answer sheets.

U6 Further Mathematics Mock

Paper 2 (Mechanics/Statistics)

PLEASE GIVE STUDENTS 2 ANSWERS BOOKS – ONE FOR MECHANICS AND ONE FOR STATISTICS

February 2020
2019-2020

Duration: 1 hour

Total number of marks: 48

Write your answers in the spaces provided.
Additional paper may be used if necessary.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Mechanics [27]

1.

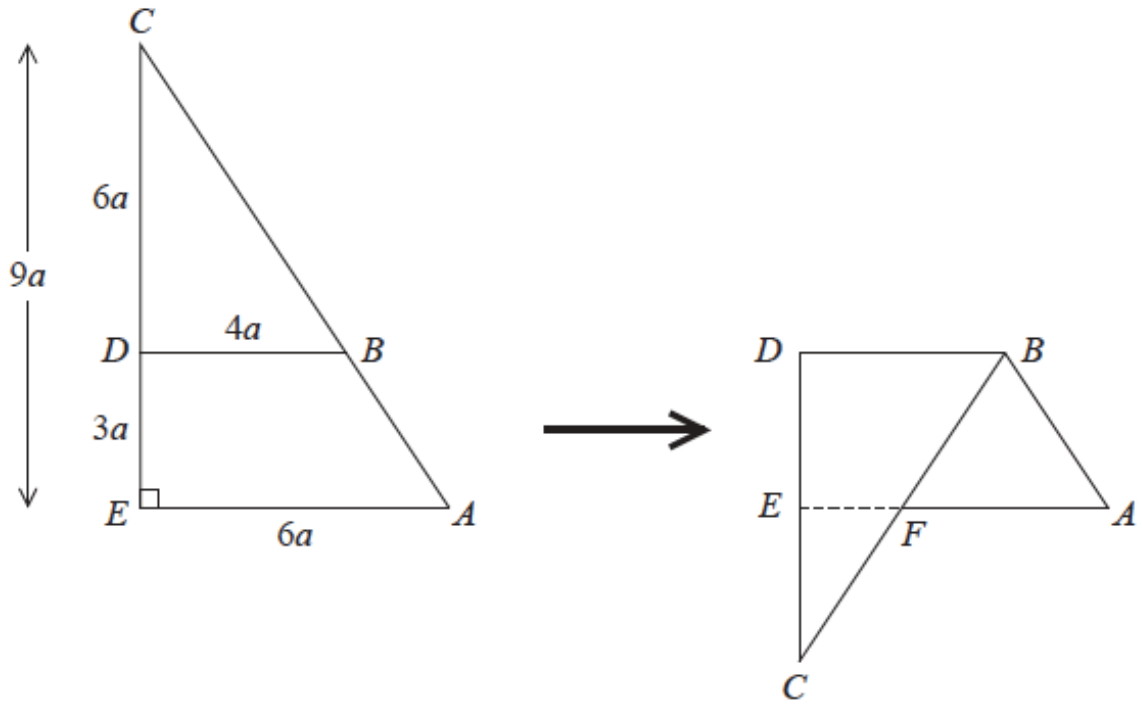

Figure 2

The uniform triangular lamina $ABCDE$ is such that angle $CEA = 90°$, $CE = 9a$ and $EA = 6a$. The point D lies on CE, with $DE = 3a$. The point B on CA is such that DB is parallel to EA and $DB = 4a$. The triangular lamina is folded along the line DB to form the folded lamina $ABDECF$, as shown in Figure 2.

The distance of the centre of mass of the triangular lamina from DC is d_1

The distance of the centre of mass of the folded lamina from DC is d_2

(a) Explain why $d_1 = d_2$

(1)

The folded lamina is freely suspended from B and hangs in equilibrium with BA inclined at an angle α to the downward vertical through B.

(b) Find, to the nearest degree, the size of angle α.

(9)

2.

Numerical (calculator) integration is not acceptable in this question.

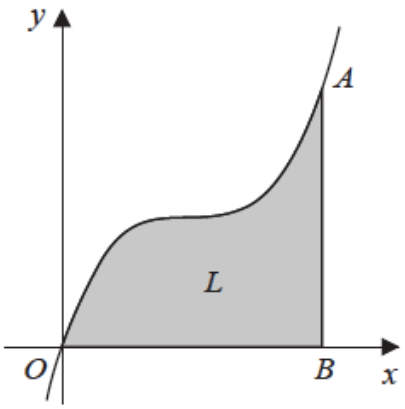

Figure 2

The shaded region OAB in Figure 2 is bounded by the x-axis, the line with equation $x = 4$ and the curve with equation $y = \frac{1}{4}(x-2)^3 + 2$. The point A has coordinates (4, 4) and the point B has coordinates (4, 0).

A uniform lamina L has the shape of OAB. The unit of length on both axes is one centimetre. The centre of mass of L is at the point with coordinates $(\bar{x}, \bar{y})$.

Given that the area of L is $8\,\text{cm}^2$,

(a) show that $\bar{y} = \frac{8}{7}$

(4)

The lamina is freely suspended from A and hangs in equilibrium with AB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(7)

3.

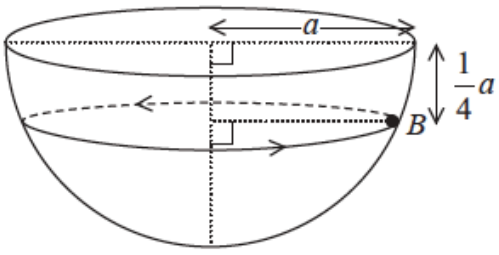

Figure 1

A hemispherical shell of radius a is fixed with its rim uppermost and horizontal. A small bead, B, is moving with constant angular speed, ω, in a horizontal circle on the smooth inner surface of the shell. The centre of the path of B is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in Figure 1.

Find the magnitude of ω, giving your answer in terms of a and g.

(6)

Statistics [21]

4.

A psychologist investigated the scores of pairs of twins on an aptitude test.
Seven pairs of twins were chosen randomly, and the scores are given in the following table.

Elder twin	65	37	60	79	39	40	88
Younger twin	58	39	61	62	50	26	84

(i) Carry out an appropriate Wilcoxon test at the 10% significance level to investigate whether there is evidence of a difference in test scores between the elder and the younger of a pair of twins. **[6]**

(ii) Explain the advantage in this case of a Wilcoxon test over a sign test. **[1]**

5.

The continuous random variable T has cumulative distribution function

$$F(t) = \begin{cases} 0 & t < 0, \\ 1 - \mathrm{e}^{-0.25t} & t \geqslant 0. \end{cases}$$

(a) Find the cumulative distribution function of $2T$. **[3]**

(b) Show that, for constant k, $\mathrm{E}(\mathrm{e}^{kt}) = \dfrac{1}{1-4k}$.

You should state with a reason the range of values of k for which this result is valid. **[7]**

(c) T is the time before a certain event occurs.

Show that the probability that no event occurs between time $T = 0$ and time $T = \theta$ is the same as the probability that the value of a random variable with the distribution $\mathrm{Po}(\lambda)$ is 0, for a certain value of λ. You should state this value of λ in terms of θ. **[4]**