U6 FM Mock (Mechanics/Statistics) 19-20 SOLUTIONS [48]

1.

(a)

In the folding process, each point of the lamina remains the same distance from *CD*	B1
	(1)

(b)

For the folded lamina: $\bar{x} = 2a \quad (= d_2)$ oe	B1

Distances from *EA*

Large triangle (*ACE*)	Removed triangle (*BCD*)	Added triangle (*BCD*)	Folded lamina
$27a^2$	$12a^2$	$12a^2$	$27a^2$
$3a$	$5a$	a	$\bar{y}$

Alternative 1

Distances from *BD*

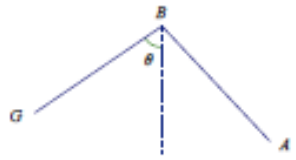
Rectangle *EDBH*	Triangle *BHA*	Triangle *DBC*	Folded lamina
$12a^2$	$3a^2$	$12a^2$	$27a^2$
$1.5a$	$2a$	$2a$	$\bar{y}$

Alternative 2

Distances from *BD*

Triangle *FAB*	Triangle *EFC*	2 x Rectangle *DGEF*	2 x Triangle *BGF*	Folded lamina
$6a^2$	$3a^2$	$12a^2$	$6a^2$	$27a^2$
$2a$	$4a$	$1.5a$	a	$\bar{y}$

Area ratios	B1
Distances from *EA*	B1
Moments about *EA*:	M1
$27 \times 3a - 12 \times 5a + 12 \times a = 27\bar{y}$	A1**ft**
$\bar{y} = \frac{11a}{9}$	A1

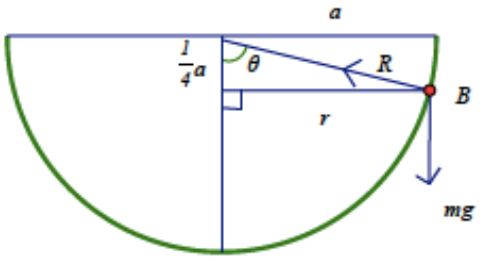
(b) cont			
	$\theta = \tan^{-1}\frac{4a-\overline{x}}{3a-\overline{y}}\left(=\tan^{-1}\frac{9}{8}\right)$ or $(90° - \theta) = \tan^{-1}$ (reciprocal)	M1	1.1b
	$\alpha = \tan^{-1}\frac{4a-\overline{x}}{3a-\overline{y}} + \tan^{-1}\frac{2}{3}$ or oe	M1	3.1b
	$= 82°$ (nearest degree)	A1	1.1b
	Alternative for the final 3 marks:		
	$\overrightarrow{BA}.\overrightarrow{BG} = \frac{2}{9}\begin{pmatrix}-9\\-8\end{pmatrix}.\begin{pmatrix}2\\-3\end{pmatrix}\left(=\frac{4}{3}\right)$	M1	1.1b
	$\cos\alpha = \frac{4/3}{\frac{2}{9}\sqrt{145}\sqrt{13}}(=0.138...)$	M1	3.1b
	$\theta = 82°$	A1	1.1b
		(9)	
			(10 marks)

2.

(a)	Total mass $= \int_0^4 (18-3x)\,dx$	M1
	$= \left[18x - \frac{3x^2}{2}\right]_0^4$	A1
	$= 18\times 4 - \frac{3\times 16}{2}\ (=72-24)\ = 48$ (kg) *	A1*
		(3)

(b)	Taking moments about the base: $\int_0^4 x(18-3x)\,dx$	M1
	$=\left[9x^2-x^3\right]_0^4 (=80)$	A1
	$\Rightarrow 48d=80$	M1
	$d=\frac{80}{48}=\frac{5}{3}$ (m)	A1
	Complete strategy	M1
	$\text{M}(A): 2T=4\cos 45°\times 4g+\frac{5}{3}\cos 45°\times 48g$	A1ft
	$\left(=\frac{96g}{\sqrt{2}}\right)$	A1ft
	$T=333$ or 330	A1
		(8)
(c)	Any appropriate comment e.g. the ball has been modelled as a point mass – its centre could be further from A	B1
		(1)
		(12

3.

		
$\updownarrow R\cos\theta = mg$	M1	3.1b
$\leftrightarrow R\sin\theta = mr\omega^2$	M1	3.3
	A1	1.1b
$\tan\theta = \dfrac{r}{a/4}$ $\quad\left(\tan\theta = \sqrt{15}\right)$	B1	1.1b
Complete strategy to find ω	M1	3.1b
$\tan\theta = \dfrac{mr\omega^2}{mg} = \dfrac{4r}{a}$, $\Rightarrow \omega^2 = \dfrac{4g}{a}$, $\omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
	(6)	
Alternative:		
$\updownarrow R\cos\theta = mg$	M1	3.1b
$\leftrightarrow R\sin\theta = ma\sin\theta\omega^2$	M1	3.3
	A1	1.1b
$\cos\theta = \frac{1}{4}$	B1	1.1b
Complete strategy to find ω	M1	3.1b
$\Rightarrow R = 4mg,\ 4mg = ma\omega^2 \Rightarrow \omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
		(6 marks)

4.

(i)		<table><tr><td>7</td><td>2</td><td>1</td><td>17</td><td>11</td><td>14</td><td>4</td></tr><tr><td>4</td><td>2</td><td>1</td><td>7</td><td>6</td><td>5</td><td>3</td></tr><tr><td>+</td><td>−</td><td>−</td><td>+</td><td>−</td><td>+</td><td>+</td></tr></table>	M1	1.1	Calculate differences, rank them and attach signs	Follow through with correct signs and ranks from incorrect differences
		H_0 : population median difference $= 0$	B1	2.5	Hypotheses correctly stated	
		H_1 : population median difference $\neq 0$				
		$P = 4+7+5+3 = 19$				
		$Q = 1+2+6 = 9$	A1	3.3	P or Q correct	
		$T = 8$	A1	3.4	Both P and Q seen, T correct	SC3: Two-sample, max 3/6
		$T_{crit} = 3$; $8 > 3$	B1	1.1	Comparison with correct T_{crit}	
		Do not reject H_0. Insufficient evidence of a difference in test scores	A1FT [6]	2.2b	Correct conclusion, in context, acknowledge uncertainty FT their T but not their T_{crit}	
(ii)		Uses magnitude of differences oe	B1 [1]	3.5b		

5.

(a)		Let H(x) be the CDF of $2T$. Then $H(x) = P(X \le x) = P(2T \le x)$ $= P(T \le \frac{1}{2}x) = F(\frac{1}{2}x)$ $= 1 - e^{-0.125x}$ [for $x \ge 0$, and 0 for $x < 0$]	M1 M1 A1 [3]	3.1a 1.1a 1.1	Convert to $P(2T \le x)$ Rearrange to get $P(T \le f(x))$ Any letter. Correct answer only, ignore other ranges	**Alternatively:** $g(T) = 2T$, $F(g^{-1}(x))$: M2 Needn't be simplified
(b)	Due to the error on the paper, all candidates get 7 marks for Q9(b). Annotate each answer with SEEN and enter 7 marks in RM. The only instance where full marks would not be awarded is where a candidate has not attempted any question. This would then need to be a 0.					
		PDF is $f(x) = 0.25e^{-0.25t}$	M1	2.1	Stated or implied	
		$E(e^{kt}) = \int_0^\infty 0.25e^{kt}e^{-0.25t}dt$	M1	1.1a	Attempt $\int e^{kt}f(t)dt$, correct limits	
		$\int_0^N 0.25e^{-(0.25-k)t}dt = \left[-\frac{0.25e^{-(0.25-k)t}}{0.25-k}\right]_0^N$	M1	2.1	Method for integration	
		$= -\frac{0.25e^{-(0.25-k)N}}{0.25-k} + \frac{0.25}{0.25-k}$	A1	1.1	Correct indefinite integral with finite upper limit	
		The first term will only converge for $k < 0.25$	B1	2.4	Consider range of k for which the result is valid	
		Then $\lim_{N\to\infty} \frac{0.25e^{-(0.25-k)N}}{0.25-k} = 0$	B1	2.1		
		$\int_0^\infty 0.25e^{-(0.25-k)t}dt$ $= \lim_{N\to\infty}\left\{-\frac{0.25e^{-(0.25-k)N}}{0.25-k} + \frac{0.25}{0.25-k}\right\}$ $= \frac{0.25}{0.25-k}$ or $\frac{1}{1-4k}$ AG	A1 [7]	2.2a	Correctly obtain given answer	
(c)		P(no event between 0 and θ) $= P(T > \theta)$ $= e^{-0.25\theta}$	M1 A1	2.1 1.1	Correct method for probability Correct formula	$1 - e^{-0.25\theta}$: M1A0
		P(0) from Po(λ) $= e^{-\lambda}$	B1	1.1	Simplified, any λ	i.e. neither 0! nor e^0 left in
		Hence same expression, with $\lambda = 0.25\theta$.	A1 [4]	2.2a	Correctly justify required result, with $\lambda = 0.25\theta$ oe stated explicitly	Need to say "same" oe