Averages and measures of spread of continuous random variables

Starter

1. **(Review of last lesson)**

The life, X, of the StayBrite light bulb is modelled by the probability density function

$f(x) = \begin{cases} ke^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$, where X is measured in thousands of hours.

(a) Find k.

(b) Find the probability that a StayBrite bulb lasts longer than 1000 hours.

(c) Find the probability that a StayBrite bulb lasts less than 500 hours.

Working: (a) $\int_0^\infty ke^{-2x}dx = 1 \quad \Rightarrow \quad \left[\frac{k}{2}e^{-2x}\right]_\infty^0 = 1$

$\frac{k}{2} - 0 = 1 \quad \Rightarrow \quad k = 2$

(b) $P(X > 1) = \int_1^\infty 2e^{-2x}dx = \left[e^{-2x}\right]_\infty^1 \approx 0.135$

The probability that a StayBrite bulb lasts longer than 1000 hours is 0.135 (3 s.f.)

(c) $P(X > 1) = \int_0^{0.5} 2e^{-2x}dx = \left[e^{-2x}\right]_{0.5}^0 = 1 - \frac{1}{e} \approx 0.632$

The probability that a StayBrite bulb lasts less than 500 hours is 0.632 (3 s.f.)

E.g. 1 The probability density function $f(x) = \begin{cases} 2x - 4 & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

Find the mean and variance.

Working: Mean $= \int_{-\infty}^\infty xf(x)dx = \int_2^3 x(2x-4)dx = \frac{8}{3}$

Variance $= \int_{-\infty}^\infty x^2 f(x)dx - \mu^2$

$= \int_2^3 x^2(2x-4)dx - \left(\frac{8}{3}\right)^2$

$= \frac{1}{18}$

E.g. 2 The EverOn torch battery has a life of X hours . The variable X is modelled by the probability density function $f(x) = \begin{cases} 3000x^{-4} & x \geq 10 \\ 0 & \text{otherwise} \end{cases}$

Find the mean and standard deviation of EverOn torch batteries.

Working: Mean $= \displaystyle\int_{-\infty}^{\infty} xf(x)dx = \int_{10}^{\infty} 3000x^{-3}dx = \left[\frac{1500}{x^2}\right]_{\infty}^{10} = 15$

Variance $= \displaystyle\int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

$= \displaystyle\int_{10}^{\infty} 3000x^{-2}dx - 15^2$

$= \left[\dfrac{3000}{x}\right]_{\infty}^{10} - 225$

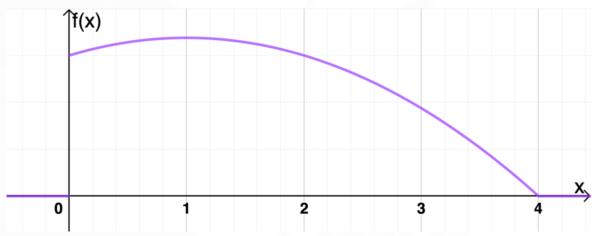
$= 75$

Standard deviation $= \sqrt{75} = 5\sqrt{3}$

E.g. 3 The crv X has pdf where $f(x) = \begin{cases} \frac{3}{80}(2+x)(4-x) & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$.

Find the mode and median.

Working: The graph indicates that the mode will be the maximum value of the curve.



Given that the roots occur when $x = -2$ and $x = 4$, the maximum will occur when $x = 1$.

So the mode is 1.

Median, $\displaystyle\int_{-\infty}^{m} f(x)dx = \frac{1}{2}$: $\quad \displaystyle\int_{0}^{m} \frac{3}{80}(2+x)(4-x)dx = \frac{1}{2}$

$\displaystyle\int_{0}^{m} (8 + 2x - x^2)dx = \frac{40}{3} \quad \Rightarrow \quad \left[8x + x^2 - \frac{1}{3}x^3\right]_0^m = \frac{40}{3}$

$24m + 3m^2 - m^3 = 40 \quad \Rightarrow \quad m^3 - 24m - 3m^2 + 40 = 0$

Since $0 \leq m \leq 4$, $m \approx 1.26$

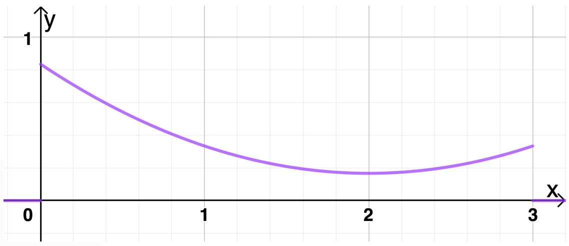
The mode is 1 and the median is 1.26 (3 s.f.).

E.g. 4 A continuous random variable is modelled by $f(x) = \begin{cases} \frac{1}{6}\left(x^2 - 4x + 5\right) & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$.

Find the mode and median.

Working:

The mode is the $x-$value that gives the curve its highest value.
So the mode is 0.



Median, $\int_{-\infty}^{m} f(x)dx = \frac{1}{2}$: $\quad \int_{0}^{m} \frac{1}{6}\left(x^2 - 4x + 5\right)dx = \frac{1}{2}$

$\int_{0}^{m} \left(x^2 - 4x + 5\right)dx = 3 \quad \Rightarrow \quad \left[\frac{1}{3}x^3 - 2x^2 + 5x\right]_0^m = 3$

$m^3 - 6m^2 + 15m = 9 \quad \Rightarrow \quad m^3 - 6m^2 + 15m - 9 = 0$

Since $0 \le m \le 3$, $m \approx 0.846$

The mode is 0 and the median is 0.846 (3 s.f.).

Video: **Mean and variance of continuous random variable**

Solutions to Starter and E.g.s

Exercise
p125 7B Qu 1i, 2i, 3-7 (8 red)