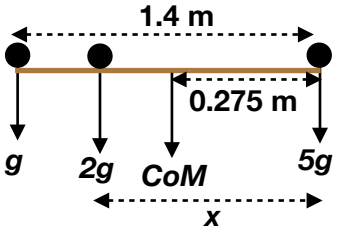
Centres of mass of standard shapes

Starter

1. **(Review of last lesson)**
A light rod of length 1.4 m has masses 1 kg and 5 kg placed at its ends. Where should a 2 kg mass be placed so that the centre of mass is 0.275 m from the 5 kg mass?



Working: Let x be the distance of the 2 kg mass from the 5 kg mass.

$\circlearrowleft$ about the 5 kg mass:

$$(1+2+5)\times 0.275 = 1\times 1.4 + 2\times x$$
$$2.2 = 1.4 + 2x$$
$$0.132 = 0.6m$$
$$m = \frac{11}{50} = 0.22$$

The 2 kg mass should be placed 0.4 m from the 5 kg mass.

2. **(Review of last lesson)**
Objects of mass m, $2m$, $3m$ and $4m$ are placed at the points $(4,\ 3)$, $(1,\ 1)$, $(3,\ 0)$ and $(2,\ -2)$ respectively. Find the centre of mass of the set of points.

Working: $\circlearrowleft$ about the origin:

$$10m\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m\begin{pmatrix}4\\3\end{pmatrix} + 2m\begin{pmatrix}1\\1\end{pmatrix} + 3m\begin{pmatrix}3\\0\end{pmatrix} + 4m\begin{pmatrix}2\\-2\end{pmatrix}$$
$$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}23\\-3\end{pmatrix}$$
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}2.3\\-0.3\end{pmatrix}$$

The centre of mass is at $\begin{pmatrix}2.3\\-0.3\end{pmatrix}$ m.

E.g. 1 Find the coordinates of the centre of mass of a uniform triangular lamina with vertices at $A(4,\ 4)$, $B(1,\ 1)$ and $C(5,\ 1)$.

Working: CoM: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{4+1+5}{3}, \frac{5+1+1}{3}\right)$

The centre of mass, G, is at $\left(\frac{10}{3}, 2\right)$

E.g. 2 A wire of length 10 cm is bent into the form of an arc whose radius is 4 cm. Find the distance of the centre of the mass to the ***wire***.

Working: Using $s = r\theta$: $\theta = \frac{10}{4} = \frac{5}{2}^{c}$

Remember to halve the angle at the centre

Using $OG = \frac{r\sin\alpha}{\alpha}$: $OG = \frac{4 \times \sin\frac{5}{4}}{\frac{5}{4}} = \frac{16}{5}\sin\frac{5}{4} \approx 3.03675$

Distance to wire $= 4 - = \frac{16}{5}\sin\frac{5}{4} \approx 0.963$

The distance of the centre of the mass to the wire is 0.963 cm.

E.g. 3 A sector of a circle has radius 8 cm and angle 60^{o}. Find the distance of the centre of the mass to the centre of the circle.

Working: ***Remember to use radians and halve the angle at the centre***

Using $OG = \frac{2r\sin\alpha}{3\alpha}$: $OG = \frac{2 \times 8 \times \sin\frac{\pi}{6}}{3 \times \frac{\pi}{6}} = \frac{16}{\pi}$ cm

The distance of the centre of the mass to the centre of the circle is $\frac{16}{\pi}$ cm.

Video: CoM (triangular laminas)
Video: CoM (sectors and semi-circles)
Video (password needed): Centre of mass of a uniform lamina
Video (password needed): Suspended objects

Solutions to Starter and E.g.s

Exercise
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