Elastic potential energy and work done

Starter

1. (Review of last lesson)

A box of weight 20 N is placed on a table. It is to be pulled along by an elastic string with natural length 15 cm and modulus of elasticity 5 N. The coefficient of friction between the box and the table is 0.4. Holding the string horizontally by its loose end, and beginning with the string just taut, how far would you have to pull before the box starts to move?

Working: $R(\uparrow)$: $R = 20$

$F_{lim} = \mu R$: $F_{lim} = 0.4 \times 20 = 8$

So there needs to be 8 N of tension in the string for the box to move.

$T = 8, \lambda = 5, l = 15$

Substituting into $T = \dfrac{\lambda x}{l}$: $8 = \dfrac{5x}{15}$

$x = 24$

You would have to pull 24 cm.

2. The work done by a variable force $f(x)$ to move from position x_1 to x_2 is given by $\int_{x_1}^{x_2} f(x)dx$. Hence, using Hooke's law $T = \dfrac{\lambda x}{l}$, calculate the work down to extend a string from x_1 to x_2.

Working:

$$\text{Work done} = \int_{x_1}^{x_2} f(x)dx$$
$$= \int_{x_1}^{x_2} \frac{\lambda x}{l}dx$$
$$= \left[\frac{\lambda x^2}{2l}\right]_{x_1}^{x_2}$$
$$= \left[\frac{\lambda x^2}{2l}\right]_{x_1}^{x_2}$$
$$= \frac{\lambda}{2l}(x_2^2 - x_1^2)$$

E.g. 1 An elastic cord has natural length 3 m and modulus of elasticity 60 N. How much work must be done to fix it between two hooks 5 m apart.

Working: Work done $= \dfrac{\lambda}{2l}(x_2^2 - x_1^2) = \dfrac{60}{2 \times 3}(2^2 - 0^2) = 40$ J

N.B. 0^2 because at the start the cord is not extended.

E.g. 2 A spring has natural length 40 cm. A force of 18 N is needed to compress it by 5 cm. How much work must be done to push it down into a box of height 15 cm?

Working: $l = 40$, when $T = 18, x = 5$

Substituting into $T = \frac{\lambda x}{l}$: $\quad 18 = \frac{5\lambda}{40}$

$$\lambda = 144$$

Make sure the units of length are metres.

Work done $= \frac{\lambda}{2l}(x_2^2 - x_1^2) = \frac{144}{2 \times 0.4}(0.25^2 - 0^2) = 11.25$ J

E.g. 3 A pinball machine fires small balls of mass 50 g by means of a spring and a light plunger. The spring and ball move in a horizontal plane. The spring has modulus of elasticity 72 N and natural length 12 cm and is compressed by 5 cm to fire a ball.

(a) Find the energy stored in the spring immediately before the ball is fired

(b) Find the speed of the ball when it is fired.

Working: (a) EPE $= \frac{\lambda x^2}{2l}$: $\quad$ EPE $= \frac{72 \times 0.05^2}{2 \times 0.12} = 0.75$

The energy stored in the spring is 0.75 J

(b) By the CoE this EPE is transferred to KE: $\quad \frac{1}{2} \times 0.05 \times v^2 = 0.75$

$$v^2 = 30$$

$$v \approx 5.48$$

The speed of the ball when it is fired is 5.48 m/s.

E.g. 4 The ceiling of a room is 2 m above the floor. A ball of mass m kg hangs from an elastic string attached to the ceiling. The natural length of the string is 0.8 m, and in equilibrium the ball rests 1 m below the ceiling. The ball is now pulled down and placed on the floor with the string stretched. Find the speed with which the ball hits the ceiling after it is released.

Working: Using Hooke's law, $T = \frac{\lambda x}{l}$: $\quad mg = \frac{0.2\lambda}{0.8} \Rightarrow \lambda = 4mg$

EPE $= \frac{\lambda x^2}{2l}$: $\quad$ EPE $= \frac{4mg \times 1.2^2}{2 \times 0.8} = \frac{18mg}{5}$

By the CoE this is transferred to KE and GPE:

$$\frac{1}{2}mv^2 + 2mg = \frac{18mg}{5}$$

$$v^2 = \frac{16g}{5}$$

$$\therefore v = \frac{28}{5} = 5.6$$

The speed at which the mass hits the ceiling is 5.6 m/s (3 s.f.)

Video (password needed): Elastic potential energy

Video: Energy stored in an elastic string or spring

Solutions to Starter and E.g.s

Exercise

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