

## Expectation and variance of the sample mean

### Starter

1. **(Review of last lesson)** Independent random variables  $X$  and  $Y$  are such that  $E(X^2) = 14$ ,  $E(Y^2) = 20$ ,  $\text{Var}(X) = 10$  and  $\text{Var}(Y) = 11$ . Find:
- the possible values of  $E(3X - 2Y)$
  - $\text{Var}(5X - 2Y)$

**Working:**

$$\begin{aligned} \text{(a)} \quad \text{Var}(X) &= E(X^2) - E^2(X): & 10 &= 14 - E^2(X) \\ & & E(X) &= \pm 2 \\ & & 11 &= 20 - E^2(Y) \\ & & E(Y) &= \pm 3 \end{aligned}$$

Both positive:  $E(3X - 2Y) = 3 \times 2 - 2 \times 3 = 0$   
 Both negative:  $E(3X - 2Y) = 3 \times (-2) - 2 \times (-3) = 0$   
 $E(X) = 2$  &  $E(Y) = -3$ :  $E(3X - 2Y) = 3 \times 2 - 2 \times (-3) = 12$   
 $E(X) = -2$  &  $E(Y) = 3$ :  $E(3X - 2Y) = 3 \times (-2) - 2 \times 3 = -12$   
 The possible values of  $E(3X - 2Y)$  are 0 and  $\pm 12$ .

$$\text{(b)} \quad \text{Var}(5X - 2Y) = 5^2 \times 10 + 2^2 \times 11 = 294$$

**E.g. 1** Find the expected value and the variance of the sample mean:

- $E(X) = 10$ ,  $\text{Var}(X) = 1.6$ ,  $n = 20$
- $X \sim N(120, 5^2)$ ,  $n = 8$
- $X \sim B(9, 0.4)$ ,  $n = 15$
- $X \sim \text{Po}(8.5)$ ,  $n = 30$
- $X \sim \text{Geo}(0.25)$ ,  $n = 32$

**Working:**

$$\begin{aligned} \text{(a)} \quad E(\bar{X}) &= E(X) = 10 \\ \text{Var}(\bar{X}) &= \frac{\text{Var}(X)}{n} = \frac{1.6}{20} = \frac{2}{25} = 0.08 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad E(\bar{X}) &= E(X) = 120 \\ \text{Var}(\bar{X}) &= \frac{\text{Var}(X)}{n} = \frac{5^2}{8} = \frac{25}{8} = 3.125 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad X &\sim B(9, 0.4) \quad \Rightarrow \quad E(X) = 9 \times 0.4 = 3.6 \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{Var}(X) = 9 \times 0.4(1 - 0.4) = 2.16 \\ E(\bar{X}) &= E(X) = 3.6 \\ \text{Var}(\bar{X}) &= \frac{\text{Var}(X)}{n} = \frac{2.16}{15} = \frac{18}{125} = 0.144 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad X &\sim \text{Po}(8.5) \quad \Rightarrow \quad E(X) = \text{Var}(X) = 8.5 \\ E(\bar{X}) &= E(X) = 8.5 \\ \text{Var}(\bar{X}) &= \frac{\text{Var}(X)}{n} = \frac{8.5}{30} = \frac{17}{60} = 0.28\bar{3} \end{aligned}$$

$$(e) \quad X \sim \text{Geo}(0.25) \quad \Rightarrow \quad E(X) = \frac{1}{0.25} = 4$$
$$\text{Var}(X) = \frac{1 - 0.25}{0.25^2} = 12$$

$$E(\bar{X}) = E(X) = 4$$
$$\text{Var}(\bar{X}) = \frac{\text{Var}(X)}{n} = \frac{12}{32} = \frac{3}{8} = 0.375$$

**E.g. 2** A machine fills cans of drink with a mean liquid content of 355 ml and standard deviation 17 ml. A sample of 30 cans is taken. Calculate the expectation and variance of the sample mean of the 30 cans.

**Working:**

$$E(\bar{X}) = E(X) = 355$$
$$\text{Var}(\bar{X}) = \frac{\text{Var}(X)}{n} = \frac{17^2}{30} = 9.6\dot{3}$$

The expectation and variance are 355 ml and 9.6 $\dot{3}$  ml<sup>2</sup>.

**E.g. 3** Bananas are sold in bags of 5 with the mass of the bag being exactly 37 g. The mass of one banana has mean mass 180 g and standard deviation 16.4 g. Find the mean and standard deviation of a bag of bananas.

**Working:**

$$E(\bar{B}) = E(B) = 180$$
$$\text{Var}(\bar{B}) = \frac{\text{Var}(B)}{n} = \frac{16.4^2}{5} = 53.792$$
$$E(5\bar{B} + 37) = 5E(\bar{B}) + 37 = 5E(B) + 37 = 5 \times 180 + 37 = 937$$
$$\text{Var}(5\bar{B} + 37) = 5\text{Var}(\bar{B}) = 5^2 \times 53.792 = 1344.8$$

Standard deviation is  $\frac{82\sqrt{5}}{5} \approx 36.7$ .

The mean and standard deviation are 937 g and 36.7 g (3 s.f.)

**Video:** [Deriving the mean and variance of the sample mean](#)  
**Video:** [Expectation and variance of the sample mean](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p155 8B Qu 1i, 2-5, (7-8 red)

**Summary**

$$E(\bar{X}) = E(X)$$
$$\text{Var}(\bar{X}) = \frac{\text{Var}(X)}{n}$$