Hooke's law

Starter

1. **(Review of last lesson)**
A particle moves under the resistive force given by $f(x) = 4x \ln 2x$ from the point where $x = 1$ to the point where $x = e$. Calculate the work done against the resistive force.

Working: Work done $= \int_1^e 4x \ln 2x \, dx$

$$= \left[2x^2 \ln 2x\right]_1^e - \int_1^e 2x \, dx \quad \textit{using integration by parts}$$

$$= \left[2x^2 \ln 2x - x^2\right]_1^e$$

$$= \left(2e^2 \ln 2e - e^2\right) - \left(2 \ln 2 - 1^2\right)$$

$$= 2e^2(\ln 2 + \ln e) - e^2 - 2 \ln 2 + 1$$

$$= 2e^2 \ln 2 + 2e^2 - e^2 - 2 \ln 2 + 1 \quad \textit{since } \ln e = 1$$

$$= 2(e^2 - 1)\ln 2 + e^2 + 1$$

E.g. 1 A light elastic string of natural length 0.7 m and modulus of elasticity 50 N has one fixed end and a particle of mass 1.4 kg attached to the other. The system hangs vertically in equilibrium. Find the extension of the string.

Working: $l = 0.7, \lambda = 50, T = 1.4g$

Substituting into $T = \frac{\lambda x}{l}$: $\quad 1.4g = \frac{50x}{0.7}$

$$x = 0.19208 \text{ m}$$

The extension of the string is 19.208 cm

E.g. 2 A set of kitchen scales consists of a light scale pan supported on a spring of natural length 8 cm. When measuring 1.5 kg of flour the spring is compressed by 7 mm. Find:
(a) the modulus of elasticity
(b) the mass of the heaviest object that can be measured if it is impossible to compress the spring by more than 15 mm.

Working: (a) $l = 80, T = 1.5g, x = 7, \lambda = ?$ ***same units for x and l***

Substituting into $T = \frac{\lambda x}{l}$: $\quad 1.5g = \frac{7\lambda}{80}$

$$\lambda = 168 \text{ N}$$

(b) Let the mass be m

$l = 80, T = mg, x = 15, \lambda = 168$ ***same units for x and l***

Substituting into $T = \frac{\lambda x}{l}$: $\quad mg = \frac{168 \times 15}{80}$

$$m = \frac{45}{14} \approx 3.21$$

The mass of the heaviest object is 3.21 kg (3 s.f.)

E.g. 3 A light spring has modulus of elasticity λ N and natural length l m. One end is attached to a ceiling, the other to a particle of mass m kg which hangs in equilibrium below the ceiling. In this situation the total length of the string is L m. When $m = 0.03, L = 0.49$, and when $m = 0.06, L = 0.53$.

(a) Find the value of l and and the value of λ.

(b) Find the value of m when $L = 0.6$.

Working: (a) $L = l + x$

When $L = 0.49, x = 0.49 - l$

When $w = 0.03, L = 0.49$: $0.03g = \dfrac{\lambda(0.49 - l)}{l}$

$0.03gl = \lambda(0.49 - l)$

When $w = 0.06, L = 0.53$: $0.06g = \dfrac{\lambda(0.53 - l)}{l}$

$0.06gl = \lambda(0.53 - l)$

Dividing the 2nd equation by the first: $\dfrac{0.06gl}{0.03gl} = \dfrac{\lambda(0.53 - l)}{\lambda(0.49 - l)}$

Cancel and cross multiply: $2(0.49 - l) = 0.53 - l$

$\therefore l = 2 \times 0.49 - 0.53 = 0.45$ m

Substituting into $0.03g = \dfrac{\lambda(0.49 - l)}{l}$: $0.03g = \dfrac{0.04\lambda}{0.45}$

$\lambda = 3.3075$ N

(b) When $L = 0.6, x = 0.6 - 0.45 = 0.15$

Substituting into $T = \dfrac{\lambda x}{l}$: $mg = \dfrac{3.3075 \times 0.15}{0.45}$

$m = 0.1125$ kg

Video (Exam solutions): Hooke's law

Video (Khan): Hooke's law

Video: Hooke's law example

Solutions to Starter and E.g.s

Exercise

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