Impulse-Momentum of Variable Forces

Starter

1. **(Review of last lesson)**
A rock of mass 10 kg falls over a cliff and drops vertically on to a field 200 m below. The air resistance is given by $0.0392v^2$ N when the speed is v m/s. Find:
(a) the terminal speed for a fall of indefinite distance
(b) the speed with which the rock hits the field

Working: (a) Using $F = ma$: $10g - 0.0392v^2 = 10a$

Since displacement is required, use $a = v\dfrac{dv}{dx}$

$$10g - 0.0392v^2 = 10v\frac{dv}{dx}$$

Separation of variables: $\displaystyle\int dx = \int \frac{v}{g - 0.00392v^2}\,dv$

$$x = -\frac{6250}{49}\ln|g - 0.00392v^2| + c$$

When $t = 0, x = 0, v = 0$: $c = \dfrac{6250}{49}\ln g$

$$x = \frac{6250}{49}\ln g - \frac{6250}{49}\ln|g - 0.00392v^2|$$

$$x = \frac{6250}{49}\ln\left|\frac{g}{g - 0.00392v^2}\right|$$

$$e^{\frac{49}{6250}x} = \frac{g}{g - 0.00392v^2}$$

$$g - 0.00392v^2 = ge^{-\frac{49}{6250}x}$$

$$0.00392v^2 = g - ge^{-\frac{49}{6250}x}$$

$$v^2 = \frac{12500}{49}g\left(1 - e^{-\frac{49}{6250}x}\right)$$

As $x \to \infty$, $v \to \sqrt{\dfrac{12500}{49}g} = 50$

The terminal velocity is 50 m/s

(b) When $x = 200$, $v = \sqrt{\dfrac{12500}{49}g\left(1 - e^{-\frac{49}{6250}\times 200}\right)} = 44.5$

The speed with which the rock hits the field is 44.5 m/s

2. **(Review of AS FM material)**
A particle of mass 3 kg is moving along a straight line in the direction AB with speed 6 m/s when a force is applied to it. After 4 seconds the particle is moving in the direction BA with speed 2 m/s. Find the magnitude and direction of the force.

Working: Momentum before $= 3 \times 6 = 18$
Momentum after $= 3 \times (-2) = -6$ ***$-$ve due to direction change***
Since the speed is decreased: Impulse $= F \times 4 = 18 - (-6)$
$\therefore F = 6$
The force is 6 N and acts in the direction BA

E.g. 1 A body of mass 1.4 kg falls from rest in a medium which exerts a resistance of $(2t + k)$ N, where k is a constant. The speed of the body after falling for 4 seconds is 18 m/s. Find:
(a) the value of k
(b) the speed after a further 3 seconds.

Working (a) Resultant force, $F = 1.4g - (2t + k) = 13.72 - 2t - k$
When $t = 0$, $v = 0$ and when $t = 4$, $v = 18$ so

$$\text{Impulse} = \int_0^4 (13.72 - 2t - k)dt = 1.4 \times 18 - 1.4 \times 0$$

$$\int_0^4 (13.72 - 2t - k)dt = 25.2$$

$$\left[13.72t - t^2 - kt\right]_0^4 = 25.2$$

$$54.88 - 16 - 4k = 25.2$$

$$k = 3.42$$

(b) Since $k = 3.42$, $F = 10.3 - 2t$
Using impulse = increase in momentum:

$$\int_0^7 (10.3 - 2t)dt = 1.4v$$

$$1.4v = \left[10.3t - t^2\right]_0^7$$

$$1.4v = 10.3 \times 7 - 7^2 - (0 - 0)$$

$$v = 16.5$$

The speed after 7 seconds is 16.5 m/s.

E.g. 2 A body of mass 2 kg moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ m/s, collides with a body of mass 3 kg, moving with velocity $(-4\mathbf{i} + 5\mathbf{j})$ m/s. After the collision the two bodies coalesce. Find the common velocity of the combined body after the impact.

Working: Let $\mathbf{v}$ be the velocity of the combined bodies after the collision.

$$(2 + 3)\mathbf{v} = 2(3\mathbf{i} + 4\mathbf{j}) + 3(-4\mathbf{i} + 5\mathbf{j})$$

$$5\mathbf{v} = -6\mathbf{i} + 23\mathbf{j}$$

$$\mathbf{v} = -1.2\mathbf{i} + 4.6\mathbf{j}$$

$(-1.2\mathbf{i} + 4.6\mathbf{j})$ m/s

The common velocity after the impact is $(-1.2\mathbf{i} + 4.6\mathbf{j})$ m/s.

E.g. 3 A particle, A, of mass 500 g, is acted on by a variable force F N, which is defined as:

$F = 0.3t^2 + 0.5t$ for $0 \le t \le 3$ s
$F = t + 1.2$ for $3 < t \le 5$ s.

Find the speed of A after 4 seconds if the initial speed is 2 m/s.

Working: The impulse needs to be calculated in 2 parts

$$I = \int_0^3 (0.3t^2 + 0.5t)dt + \int_3^4 (t + 1.2)dt = \ldots = 9.65 \text{ Ns}$$

Impulse = change in momentum

$$9.65 = 0.5v - 0.5 \times 2$$

So $v = 21.3$ m/s

Video (password needed): Work done by a variable force

Solutions to Starter and E.g.s

Exercise
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