Impulsive tension in strings

Starter

1. **(Review of last lesson)** Two smooth spheres, A of mass m and B of mass $3m$, are free to move on a horizontal table. A is projected towards B, which is at rest, and strikes B with speed $2u$. On impact the line through their centres makes an angle of 60° with the velocity of A before impact. If the coefficient of restitution is $\frac{1}{3}$ find, in terms of u, the magnitude and direction of the velocity of each sphere just after impact.

Working: $\perp$ to the line of centres (velocity components are unchanged):

Velocity of $A = 2u \sin 60 = u\sqrt{3}$

Velocity of $B = 0$

Let the velocities of A and B in the direction of the line of centres after impact be v_A and v_B respectively.

Conservation of momentum:

$$2mu \cos 60 = mv_A + 3mv_B$$
$$v_A + 3v_B = u$$

Newton's law of impact:

$$v_B - v_A = \frac{1}{3} \times 2u \cos 60$$
$$-3v_A + 3v_B = u$$

Solving simultaneously: $v_A = 0$ $\quad v_B = \frac{1}{3}$

Velocity of A is $u\sqrt{3}$ perpendicular to the line of centres

Velocity of B is $\frac{1}{3}u$ parallel to the line of centres

E.g. 1 Two particles A and B, joined by light inextensible string, are lying together on a smooth horizontal plane. The masses of A and B are 1 kg and 1.5 kg respectively. A is projected away from B with a speed of 5 m/s. Find

(a) the speed of each particle after the string jerks taut,

(b) the impulse in the string when it jerks taut.

Working: (a) Momentum before $= 1 \times 5 = 5$

Momentum after $= (1 + 1.5)v$

$(1 + 1.5)v = 5$

$v = 2$ m/s

The speed of each particle after the string jerks taut is 2 m/s.

(b) ***Either*** Impulse = change in momentum for B

= final momentum $-$ initial momentum

$= 1.5 \times 2 - 0 = 3$ Ns

or Impulse = change in momentum for A

= final momentum $-$ initial momentum

$= 1 \times 2 - 1 \times 5 = -3$ Ns

N.B. The negative sign for A shows the impulse is acting in the opposite direction

E.g. 2 Two particles A and B of equal mass m are connected by a light inextensible string of length l. Initially they are held at rest in the air, side by side. A is then released from rest.

(a) Find, in terms of l and g, A's speed just as the string is about to jerk taut.

(b) If B is released at this instant find, in terms of l and g, the common speed with which A and B together begins to move.

(c) Find the impulse in the string.

Working:

(a) A falls freely under gravity

Taking down as positive: $u = 0, s = l, a = 9.8, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 0^2 + 2gl$

$$v = \sqrt{2gl}$$

A's speed just as the string is about to jerk taut is $\sqrt{2gl}$

(b) Let the velocities of A and B when the string goes taut be V

Conservation of momentum: $m\sqrt{2gl} = mV + mV$

$$V = \frac{1}{2}\sqrt{2gl}$$

The common speed is $\frac{1}{2}\sqrt{2gl}$.

(c) ***Either*** Impulse = change in momentum for B

= final momentum − initial momentum

$$= -m \times \frac{1}{2}\sqrt{2gl}$$

or Impulse = change in momentum for A

= final momentum − initial momentum

$$= m\sqrt{2gl} - m \times \frac{1}{2}\sqrt{2gl}$$

$$= \frac{1}{2}m\sqrt{2gl}$$

The impulse in the string is $\frac{1}{2}m\sqrt{2gl}$ Ns

E.g. 3 Two particles, A and B, of masses 1.5 kg and 2.5 kg respectively are attached by a light, slack in extensible string and lie on a smooth horizontal surface. Particle A is initially at rest while particle B is projected at a speed of $4\sqrt{2}$ m/s and when the string becomes taut it makes an angle of 45^o with the string. Find:

(a) the speed of A just after the string is taut

(b) the speed and direction of B just after the string is taut

(c) the impulse in the string

Working:

(a) CoM parallel to string: $2.5 \times 4\sqrt{2}\cos 45 = (1.5 + 2.5)v$

Velocity of A is $v = 2.5$ m/s

The speed of A just after the string is taut is 2.5 m/s.

(b) For B: Velocity parallel to string is 2.5 m/s

Velocity perpendicular to string is the same as before the string became taut i.e. $4\sqrt{2}\sin 45 = 4$

So velocity of B is $\sqrt{4^2 + 2.5^2} = 4.717$ m/s

Direction is $\tan^{-1}\dfrac{4}{2.5} = 58.0^o$ to line of the string

(c) Impulsive tension for A = $1.5 \times 2.5 - 0 = 3.75$ Ns

E.g. 4 A light inextensible string has particles A and B, each of mass 400 g, attached to either end. The string and particles rest on a smooth table. The particle at A is hit with an impulse of 8 Ns at an angle 30^o to the string, which is straight but not taut and at rest. What is the velocity of B after the impulse at the instant the string becomes taut?

Working:

Let u be the initial speed of A and v be the speed of A and B when the string becomes taut.

For A: Impulse = change in momentum: $0.4u = 8$

$u = 20$

CoM parallel to string: $0.4 \times 20\cos 30 = (0.4 + 0.4)v$

$v = 5\sqrt{3}$ m/s

The speed of B just after the string is taut is $5\sqrt{3}$ m/s along BA

Solutions to Starter and E.g.s

Exercise

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