Linear combinations of normal variables

Starter

1. (Review of last lesson) Find the best estimates of the population mean and standard deviation from this sample. Give your answers to 3 s.f..

Weight	$50 \leq x < 55$	$55 \leq x < 60$	$60 \leq x < 70$	$70 \leq x < 80$	$80 \leq x < 85$
Frequency	23	28	61	54	19

Working:

$\bar{x} \approx 67.027$
$\sigma^2 x \approx 84.472$
$\sigma x \approx 9.191$
$s^2 x \approx 84.931$
$s x \approx 9.216$
The unbiased estimates for the population mean and standard deviation are 67.0 (5 s.f.) and 9.21 (4 s.f.) respectively.

2. Given that $X \sim$ N$(12, 1.4)$, find the value of:
(a) $P(X > 13)$
(b) $P(X \leq 10.6)$
(c) $P(11.2 \leq X \leq 12.5)$
(d) a such that $P(X \geq a) = 0.63$

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E.g. 1 Let X and Y be independent random variables such that $X \sim$ N$(48, 25)$ and $Y \sim$ N$(185, 144)$. Find:
(a) $P(X + Y > 230)$
(b) $P(Y - X \leq 125)$
(c) $P(238 < 2Y - 3X \leq 245)$
(d) the value of k such that $P(5X + Y \leq k) = 0.79$

Working:

(a) $X + Y \sim$ N$(48 + 185, 25 + 144) =$ N$(233, 13^2)$
$P(X + Y > 230) = 0.591$ (3 s.f.)

(b) $Y - X \sim$ N$(185 - 48, 25 + 144) =$ N$(137, 13^2)$
$P(Y - X \leq 125) = 0.178$ (3 s.f.)

(c) $2Y - 3X \sim$ N$(370 - 144, 2^2 \times 144 + 3^2 \times 25) =$ N$(226, 801)$
$P(238 < 2Y - 3X \leq 245) = 0.0848$ (3 s.f.)

(d) $5X + Y \sim$ N$(240 - 185, 5^2 \times 25 + 144) =$ N$(55, 769)$
$P(5X + Y \leq k) = 0.79 \quad \Rightarrow \quad k \approx 77.363$
The value of k is 77.4 (3 s.f.).

E.g. 2 Each Tuesday, Sally leaves home to play tennis. The time she takes to travel to and from the tennis courts is normal distributed with mean of 50 minutes and standard deviation of 10 minutes. The length of her match is also a normal variable with mean 62 minutes and standard deviation 13 minutes. Find the probability that:

(a) Sally is away from home for more than 2 hours

(b) Sally spends more time travelling than playing tennis.

Give your answers to 3 s.f..

Working: (a) Let T be the random variable "travel time, in minutes, to and from the courts" and M be the random variable "time taken, in minutes, by the match".

i.e. $T \sim \mathrm{N}(50,\ 10^2)$ and $M \sim \mathrm{N}(62,\ 13^2)$

So $T + M \sim \mathrm{N}(50 + 62,\ 10^2 + 13^2) = \mathrm{N}(112,\ 269)$

$P(T + M > 120) = 0.31286$

The probability that Sally is away from home for more than 2 hours is 0.313 (3 s.f.)

(b) $M - T \sim \mathrm{N}(62 - 50,\ 10^2 + 13^2) = \mathrm{N}(12,\ 269)$

$P(M - T < 0) \approx 0.23219$

The probability that Sally spends more time travelling than playing tennis is 0.232.

E.g. 3 Let X be a random variable such that $X \sim \mathrm{N}(35,\ 4^2)$. If 3 independent observations of X are made, find $P(\bar{X} > 34)$ to 3 s.f..

Working:

$E(\bar{X}) = 35$

$\mathrm{Var}(\bar{X}) = \dfrac{\mathrm{Var}(X)}{n} = \dfrac{4^2}{3} = \dfrac{16}{3}$

$\bar{X} \sim \mathrm{N}\left(35,\ \dfrac{16}{3}\right)$

$P(\bar{X} > 34) \approx 0.667497 = 0.667$ (3 s.f.)

E.g. 4 Let Y be a random variable such that $Y \sim \mathrm{N}(290,\ 108)$. If 9 independent observations of Y are made, find $P(\bar{Y} \leq 295)$ to 3 s.f..

Working:

$E(\bar{Y}) = 290$

$\mathrm{Var}(\bar{X}) = \dfrac{\mathrm{Var}(X)}{n} = \dfrac{108}{9} = 12$

$\bar{X} \sim \mathrm{N}(290,\ 12)$

$P(\bar{Y} \leq 295) \approx 0.925542 = 0.926$ (3 s.f.)

Video: Linear combinations of normal distributions

Solutions to Starter and E.g.s

Exercise

p159 8D Qu 1i, 2-8 (9-11 red)