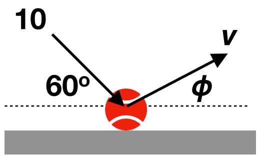
Oblique collisions of two objects

Starter

1. **(Review of last lesson)** A smooth sphere of mass 5 kg strikes a wall with speed 10 m/s at an angle of 60^o to the wall. The impulse on the sphere is 60 Ns. Find, in any order:
   (a) the rebound speed
   (b) the angle of rebound
   (c) the coefficient of restitution.



Working: Speed // to the plane:

$$10\cos 60 = v\cos\phi \qquad \text{— (1)}$$

Speed $\perp$ to the plane:

$$e \times 10\sin 60 = v\sin\phi \qquad \text{— (2)}$$

Considering impulse (i.e. momentum $\perp$ to the wall):

$$5 \times 10\sin 60 + 5v\sin\phi = 60 \qquad \text{— (3)}$$

Replacing $v\sin\phi$ by $e \times 10\sin 60$:

$$5 \times 10\sin 60 + 5e \times 10\sin 60 = 60$$
$$e \times 10\sin 60 = 12 - 10\sin 60$$
$$e \approx 0.3856$$

Dividing equation (2) by equation(1):

$$\tan\phi = e\tan 60$$
$$\phi \approx 33.74^o$$

Substituting in (1) to find v: $10\cos 60 = v\cos 33.74$

$$v \approx 6.012$$

(a) The rebound speed is 6.01 m/s (3 s.f.)
(b) The angle of rebound is 33.7^o (3 s.f.)
(c) The coefficient of restitution is 0.386 (3 s.f.)

2. **(Review of AS FM material)**
   In a game of snooker the white ball, moving at 2 m/s, strikes the pink full on. The pink starts to move with a speed of 1.6 m/s. Given that the balls have the same mass, find:
   (a) the speed of the white ball after impact and,
   (b) the coefficient of restitution, e.

Working: (a) Let the mass of the balls be m.
By conservation of momentum: $2m = mv + 1.6m$

$$v = 0.4$$

The speed of the white ball after impact is 0.4 m/s

(b) $e \times$ approach $=$ separation
Considering the white wall: $e \times 2 = 1.2 - 0.4$

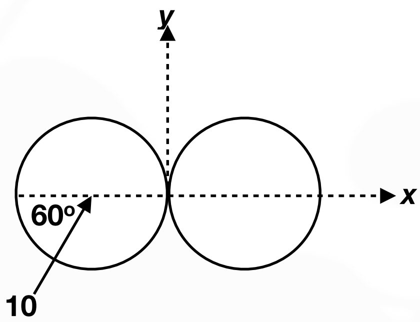
$$e = 0.6$$

The coefficient of restitution is 0.6.

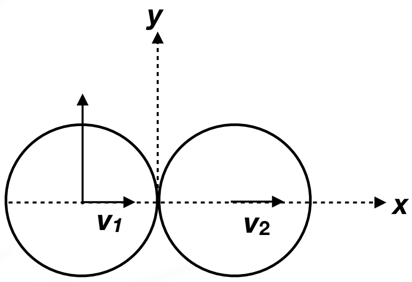
E.g. 1 In a game of shove-ha'penny, a coin is moving along the board at a speed of 10 m/s when it clips a second coin, of the same mass, at an angle. At the instant when the coins come into contact, the first coin is moving at 60^{o} to the line of centres. The coefficient of restitution is 0.6. Find how the two coins are moving immediately after the collision.
Hint: draw the line of centres across the page (the line of centres becomes the $x-$axis).

Working: The impulse acts along the line of centre. Therefore, the 2nd coin will move away along the $x-$axis. Let v_1 be the speed component of the 1st coin and let v_2 be the speed of the 2nd coin in the $x-$direction.
The $x-$ and $y-$components of the velocity are calculated separately.

Before the collision



After the collision



$x-$***direction (parallel to the line of centres):***

Conservation of momentum: $10m\cos 60 = mv_1 + mv_2$

$\therefore v_1 + v_2 = 5$

Newton's law of impact: $v_2 - v_1 = e \times 10\cos 60$

$-v_1 + v_2 = 3$

Solving simultaneously gives: $v_1 = 1$ $v_2 = 3$

$y-$***direction (perpendicular to the line of centres):***

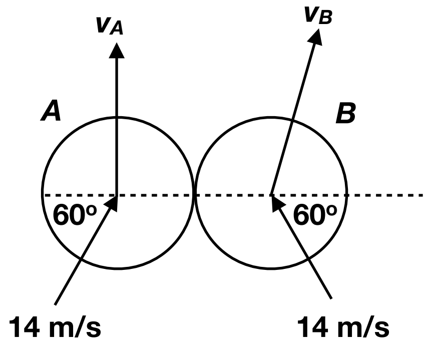
velocity component before = velocity component after

1st coin: $10\sin 60 = 5\sqrt{3}$

2nd coin: 0

After the collision, the first coin has velocity $\begin{pmatrix} 1 \\ 5\sqrt{3} \end{pmatrix}$ and the 2nd coin has velocity $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

E.g. 2 Two smooth spheres, A and B, of equal radius, have the same speed of 14 m/s immediately before they collide. The mass of A is 0.8 kg and the mass of B is 0.7 kg. Before the collision the path of each sphere makes an angle of 60° with the line of centres, and immediately after the collision A moves perpendicular to the line of centres (see diagram).
Calculate:
(a) the speed of A immediately after the collision
(b) the coefficient of restitution.



Working: (a) Speed $\perp$ to the line of centres remains the same:

$$v_A = 14 \sin 60 = 7\sqrt{3} \approx 12.12$$

The speed of A immediately after the collision is 12.1 m/s.

(b) Parallel to the line of centres:

Conservation of momentum:

$$0.8 \times 14 \cos 60 - 0.7 \times 14 \cos 60 = 0.7 v_B \cos\theta$$
$$v_B \cos\theta = 1$$

Newton's law of impact: $v_B \cos\theta = e(14 \cos 60 + 14 \cos 60)$

$$v_B \cos\theta = 14e$$

Equating the equations: $14e = 1 \quad \Rightarrow \quad e = \dfrac{1}{14}$

The coefficient of restitution is $\dfrac{1}{14}$

E.g. 3 Two smooth spheres, A and B, with equal radii, lie on a horizontal plane. The mass of B is twice that of A. The spheres are projected towards each other and they collide when the line joining their centres is in the direction of the unit vector $\mathbf{i}$. The velocity vectors of A and B just before impact are $(2\mathbf{i} + \mathbf{j})$ m/s and $(\mathbf{i} - \mathbf{j})$ m/s respectively. If $e = \dfrac{1}{2}$, find:

(a) their velocity vectors just after impact
(b) the kinetic energy lost due to the collision

Working: (a) Since the line of centres is in the $\mathbf{i}-$direction, the velocity components in the $\mathbf{j}-$direction are unchanged.
Let the mass of A be m so the mass of B is $2m$.
Let the velocities of A and B in the $\mathbf{i}-$direction after impact be v_A and v_B respectively.

Conservation of momentum:

$$2m + 2m = m v_A + 2m v_B$$
$$v_A + 2v_B = 4$$

Newton's law of impact:

$$v_B - v_A = \frac{1}{2}(2 - 1)$$
$$-2v_A + 2v_B = 1$$

Solving simultaneously: $v_A = 1 \qquad v_B = \dfrac{3}{2}$

The velocity vectors just after impact are A: $(\mathbf{i} + \mathbf{j})$ m/s and B: $\left(\dfrac{3}{2}\mathbf{i} - \mathbf{j}\right)$ m/s.

(b) Since the velocity component in the $\mathbf{j}-$direction are unchanged, only the velocity in the $\mathbf{i}-$direction needs to be considered.

KE lost = KE before − KE after

$$= \frac{1}{2}m \times 2^2 + \frac{1}{2} \times 2m \times 1^2 - \frac{1}{2}m \times 1^2 - \frac{1}{2} \times 2m \times \left(\frac{3}{2}\right)^2$$

$$= 2m + m - \frac{1}{2}m - \frac{9}{4}m$$

$$= \frac{1}{4}m$$

The kinetic energy lost due to the collision is $\frac{1}{4}m$

Video (password needed): Oblique impact with a stationary sphere

Video (password needed): Oblique impact with a moving sphere

Video (password needed): Loss of kinetic energy and deflection in oblique impacts

Solutions to Starter and E.g.s

Exercise

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