Oblique impacts with walls

Starter

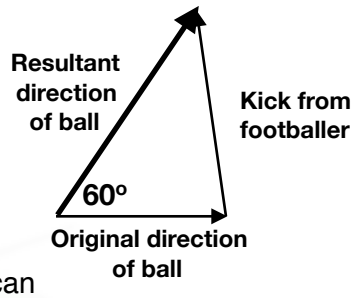
1. **(Review of last lesson)** Two balls, A and B, of masses 1.2 kg and 0.8 kg respectively, are rolling on a smooth horizontal floor when they collide. Initially their velocities are $(4.2\mathbf{i} + 2.6\mathbf{j})$ m/s and $(1.2\mathbf{i} - 0.4\mathbf{j})$ m/s respectively. After the collision the velocity of A is $(1.4\mathbf{i} + 1.8\mathbf{j})$ m/s. Find the velocity of B after the collision.

Working Let $\mathbf{v}$ be the velocity of ball B after the collision.

$$0.8\mathbf{v} + 1.2(1.4\mathbf{i} + 1.8\mathbf{j}) = 1.2(4.2\mathbf{i} + 2.6\mathbf{j}) + 0.8(1.2\mathbf{i} - 0.4\mathbf{j})$$
$$0.8\mathbf{v} = 4.32\mathbf{i} + 0.64\mathbf{j}$$
$$\mathbf{v} = 5.4\mathbf{i} + 0.8\mathbf{j}$$

The common velocity after the impact is $(5.4\mathbf{i} + 0.8\mathbf{j})$ m/s.

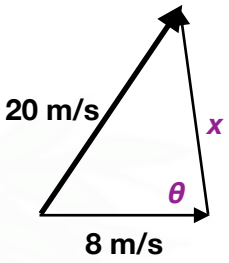
2. When a footballer receives the ball it is moving at 8 m/s. She kicks it so that its direction is diverted through 60^o and its speed is increased to 20 m/s.
In what direction does the player kick the ball?



Hint: Use the diagram to help you.

Working Since the mass of the ball does not change, we can consider velocity rather than momentum.
Use the cosine rule to find 3rd side in the triangle

$$x^2 = 20^2 + 8^2 - 2 \times 20 \times 8 \cos 60$$
$$x = \sqrt{304} = 4\sqrt{19}$$



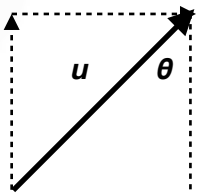
Use the sine rule to find θ

$$\frac{\sin\theta}{20} = \frac{\sin 60}{4\sqrt{19}}$$
$$\theta \approx 83.41^o$$

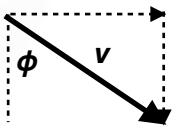
The footballer kicks the the ball at 83.4^o to the original direction of motion.

E.g. 1 Copy the diagrams and write down the components of velocity parallel and perpendicular to the wall.

Before impact



After impact



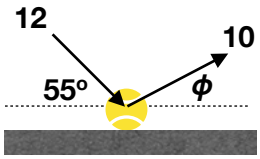
Working:

	Before impact	After impact
Parallel to wall:	$u \sin\theta$	$v \sin\phi$
Perpendicular to the wall:	$u \cos\theta$	$-v \cos\phi$

E.g. 2 A smooth sphere of mass m strikes a fixed plane surface with speed 12 m/s at an angle 55^o to the plane and rebounds with speed 10 m/s. Find:

(a) the angle at which it rebounds

(b) the coefficient of restitution



Working

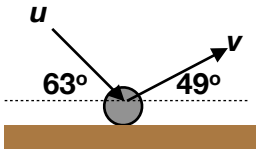
(a) Speed // to plane remains the same:

$$10\cos\phi = 12\cos 55$$
$$\phi = 46.5^o \text{ (3 sf)}$$

(b) Speed $\perp$ to the plane (use e):

$$e \times 12\sin 55 = 10\sin 46.5$$
$$e = 0.738 \text{ (3 s.f.)}$$

E.g. 3 In ice-hockey the playing area is bounded by a vertical wooden barrier. A puck strikes the barrier at an angle of 63^o and rebounds at an angle of 49^o. Calculate the coefficient of restitution.



Working

Speed // to the plane remains the same:

$$u\cos 63 = v\cos 49$$

Speed $\perp$ to the plane (use e):

$$e \times u\sin 63 = v\sin 49$$

Dividing the equations gives:

$$\frac{e \times u\sin 63}{u\cos 63} = \frac{v\sin 49}{v\cos 49}$$
$$\therefore e\tan 63 = \tan 49$$
$$e \approx 0.5861$$

The coefficient of restitution is 0.586 (3 s.f.)

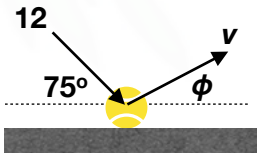
E.g. 4 A ball of mass 0.2 kg moving at 12 m/s hits a smooth horizontal wall at an angle of 75^o to the horizontal. The coefficient of restitution is 0.5. Find:

(a) the speed of the ball, v, as it leaves the wall

(b) the impulse on the ball

(c) the impulse on the plane

(d) the kinetic energy lost by the ball.



Working

(a) Speed // to the plane remains the same:

Velocity $= 12\cos 75$ (no change)

Speed $\perp$ to the plane (use e):

Velocity $= 0.5 \times 12\sin 75 = 6\sin 75$

$$v = \sqrt{(12\cos 75)^2 + (6\sin 75)^2} = 6.58 \text{ m/s (3 s.f.)}$$

(b) Impulse is the change in momentum ***perpendicular*** to the wall.

Impulse = final momentum $-$ initial momentum

$$I = 0.2 \times 6\sin 75 - 0.2 \times (-12\sin 75)$$
$$I = 3.6\sin 75 = 3.48 \text{ Ns perpendicular to the wall}$$

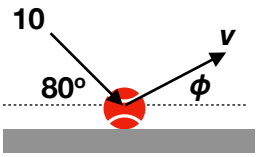
(c) By Newton's 3rd law, impulse on the plane $= 3.48$ Ns (3 s.f.)

(d) KE lost $= \frac{1}{2} \times 0.2 \times 12^2 - \frac{1}{2} \times 0.2 \times 6.58^2 = 10.1$ J (3 s.f.)

E.g. 5 A ball of mass 0.1 kg moving at 10 m/s hits a smooth horizontal plane at an angle of 80^o to the horizontal. The coefficient of restitution is 0.6. Find:

(a) the impulse on the ball

(b) the kinetic energy lost by the particle



Working (a) Speed // to the plane remains the same:

$$10\cos 80 = v\cos\phi$$

Speed $\perp$ to the plane (use e):

$$0.6 \times 10\sin 80 = v\sin\phi$$

Dividing the equations: $\tan\phi = 0.6\tan 80$

$$\phi \approx 73.62$$

Substituting to find v: $10\cos 80 = v\cos 73.62$

$$v \approx 6.159$$

For impulse, consider the change in momentum $\perp$ to the wall

$$\text{Impulse} = 0.1 \times 10\sin 80 - 0.1 \times (-6.159) \times \sin 73.62$$
$$\approx 1.576$$

The impulse is 1.58 Ns (3 s.f.)

(b) KE lost $= \frac{1}{2} \times 0.1 \times 10^2 - \frac{1}{2} \times 0.1 \times 6.159^2 = 3.10$ J (3 s.f.)

Video (password needed): Oblique impact with a fixed surface

Video (password needed): Loss of kinetic energy in an oblique impact

Video (password needed): Successive oblique impacts with plane surfaces

Solutions to Starter and E.g.s

Exercise

p210 8B Qu 1-4 (Red 5-11)