Problem solving involving work, energy and power

Starter

1. (Review of last lesson)
In a pop-gun a cork of mass 4 grams is shot out of the barrel by the release of a spring, which is compressed through a distance of 5 cm. A force of $6x$ N is needed to keep the spring compressed by x cm. Find the speed at which the cork leaves the barrel.
Hint: be careful with units.

Working: Using Hooke's law, $T = \frac{\lambda x}{l}$: $6x = \frac{\lambda x}{100l} \Rightarrow \lambda = 600l$

EPE $= \frac{\lambda x^2}{2l}$: EPE $= \frac{600l \times 0.05^2}{2l} = \frac{3}{4}$

By the CoE this is transferred to KE: $\frac{1}{2} \times 0.004 \times v^2 = \frac{3}{4}$

$v^2 = 375$

$\therefore v = 5\sqrt{15} \approx 19.4$

The speed at which the cork leaves the barrel is 19.4 m/s (3 s.f.)

2. One end of a light elastic string , of natural length 1.5 m and modulus of elasticity 30 N, is attached to a fixed point A of an inclined plane. The other end of the string is attached to a particle P of weight 25 N which lies on the plane. The plane makes an angle α with the horizontal, where $\tan\alpha = \frac{3}{4}$, and the string is parallel to a line of greatest slope of the plane. The coefficient of friction between P and the plane is 0.15.
(a) P is in equilibrium and the extension of the string is x m. Find the greatest and least value of x.
(b) P is released from rest in the position where $AP = 2$ m. By considering energy and work, find the distance AP when P first comes to rest.

Working: (a) $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$ & $\cos\alpha = \frac{4}{5}$

The particle could be just about to move up or down the plane.

About to move up the plane $\Rightarrow$ friction acts down the plane

Let T_{up} be the tension in the string.

$R(\perp)$: $R = 25\cos\alpha = 20$

$F_{lim} = \mu R$: $F_{lim} = 0.15 \times 20 = 3$

$R(//)$: $T_{up} = 25\sin\alpha + 3 = 18$

Using $T = \frac{\lambda x}{l}$: $\frac{30x}{1.5} = 18 \Rightarrow x = 0.9$

About to move down the plane $\Rightarrow$ friction acts up the plane

Let T_{down} be the tension in the string.

$R(//)$: $T_{down} = 25\sin\alpha - 3 = 12$

Using $T = \frac{\lambda x}{l}$: $\frac{30x}{1.5} = 12 \Rightarrow x = 0.6$

The greatest and least values of x are 0.9 m and 0.6 m.

(b) When $AP = 2$ m, the extension of the string is 0.5 m
Since $0.5 < 0.6$, the particle will move down the plane.
Let d be the distance the particle moves down the plane.
Lost GPE = WD against friction + WD extending the string
Lost GPE $= mgh = 25d \sin\alpha = 15d$
Work done against friction $= 3d$ since $F_{lim} = 3$
The string is extended from 0.5 m to $(d + 0.5)$ m

Work done in extending string $= \dfrac{\lambda}{2l}(x_2^2 - x_1^2)$

$$= \frac{30}{2 \times 1.5}((d + 0.5)^2 - 0.5^2)$$

$$= 10(d^2 + d)$$

Lost GPE = WD against friction + WD extending the string

$$15d = 3d + 10(d^2 + d)$$

$$d = 0.2$$

The distance AP when P first comes to rest is 2.2 m

AMSP Notes (password needed): Variable force (work, energy and impulse)

AMSP (password needed): Integral Exercise Level 1
AMSP (password needed): Integral Solutions Level 1
AMSP (password needed): Integral Exercise Level 2
AMSP (password needed): Integral Solutions Level 2

Solutions to Starter and E.g.s

Exercise
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