Review of AS complex numbers

Starter

1. (Review of last lesson) Three planes have the equations:

$\pi_1: \ x + 3y - 2z = 7 \qquad \pi_2: \ 2x - 2y + z = 3 \qquad \pi_3: \ 3x + y - z = k$

(a) Explain how you know that none of the planes are parallel to one another.

(b) Investigate the intersection of the planes when: (i) $k = 10$ (ii) $k = 12$. State the geometrical configuration of the planes in each case.

(c) For the case where the equations are consistent find the equation of the line where the planes meet.

Working: (a) The coefficients are not in the same ratio.

(b) (i)

$$\pi_1: \ x + 3y - 2z = 7$$
$$\pi_2: \ 2x - 2y + z = 3$$
$$\pi_3: \ 3x + y - z = k$$

Eliminate z***:***

$\pi_1 - 2 \times \pi_3$: $\quad -5x + y = 7 - 2k$

$\pi_2 + \pi_3$: $\quad 5x - y = 3 + k$

When $k = 10$, the equations are:

$$-5x + y = -13$$
$$5x - y = 13$$

Since these equations are multiples, the equations are consistent so the planes form a sheaf.

(ii) When $k = 12$, the equations are:

$$-5x + y = -17$$
$$5x - y = 15$$

Since these equations are not multiples, the equations are inconsistent so the planes form a triangular prism.

(c) $5x - y = 13 \qquad \pi_1: \ x + 3y - 2z = 7$

Let $x = \lambda$: $\quad y = -13 + 5\lambda$

Substitute in π_1: $\quad \lambda + 3(-13 + \lambda) - 2z = 7$

$$z = -23 + 2\lambda$$

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda \\ -13 + 5\lambda \\ -23 + 2\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ -13 \\ -23 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$$

E.g. 1 Given that $\mathrm{i} = \sqrt{-1}$, find (a) i^2 (b) i^3 (c) i^4 (d) i^5

Working: (a) $\mathrm{i}^2 = -1$ (b) $\mathrm{i}^3 = -\mathrm{i}$

(c) $\mathrm{i}^4 = 1$ (d) $\mathrm{i}^5 = \mathrm{i}$

E.g. 2 Let $z_1 = 2 + \mathrm{i}$, $z_2 = 2 - \mathrm{i}\sqrt{3}$ and $z_3 = \mathrm{i} - 1$.

(a) Write down the complex conjugates of z_1, z_2 and z_3.

(b) Express in the form $a + \mathrm{i}b$: (i) $z_2 \times z_3$ (ii) $z_3 \div z_1$

(c) Find the modulus and argument of z_1, z_2 and z_3.

(d) Express z_1, z_2 and z_3 in modulus-argument form.

Working:

(a) $z_1^* = 2 - \mathrm{i}$ $\quad z_2^* = 2 + \mathrm{i}\sqrt{3}$ $\quad z_3^* = -1 - \mathrm{i}$

(b) (i) $z_2 \times z_3 = (2 - \mathrm{i}\sqrt{3})(-1 + \mathrm{i}) = (\sqrt{3} - 2) + \mathrm{i}(\sqrt{3} + 2)$

(ii) $z_3 \div z_1 = \dfrac{-1 + \mathrm{i}}{2 + \mathrm{i}} = \dfrac{-1 + \mathrm{i}}{2 + \mathrm{i}} \times \dfrac{2 - \mathrm{i}}{2 - \mathrm{i}} = \dfrac{-1 + 3\mathrm{i}}{5}$

(c) $|z_1| = |2 + \mathrm{i}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$|z_2| = |2 - \mathrm{i}\sqrt{3}| = \sqrt{2^2 + (-\sqrt{3})^2} = \sqrt{7}$

$|z_3| = |\mathrm{i} - 1| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\text{Arg } z_1 = \tan^{-1}\dfrac{1}{2} = 0.464^c$

$\text{Arg } z_2 = 2\pi - \tan^{-1}\dfrac{\sqrt{3}}{2} = 5.569^c$ or $\text{Arg } z_2 = -\tan^{-1}\dfrac{\sqrt{3}}{2} = -0.714^c$

$\text{Arg } z_3 = \pi - \tan^{-1}\dfrac{1}{1} = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

(d) $z_1 = \sqrt{5}(\cos 0.464^c + \mathrm{i}\sin 0.464^c)$

$z_2 = \sqrt{7}(\cos(-0.714^c) + \mathrm{i}\sin(-0.714^c))$

$\quad = \sqrt{7}(\cos 0.714^c - \mathrm{i}\sin 0.714^c)$

$z_3 = \sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

E.g. 3 Express the following in the form $a + \mathrm{i}b$.

(a) $2\sqrt{3}\left[\cos\left(\dfrac{\pi}{3}\right) + \mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right]$ (b) $8(\cos 0 + \mathrm{i}\sin 0)$

(c) $2\sqrt{2}\left[\cos\left(-\dfrac{3\pi}{4}\right) + \mathrm{i}\sin\left(-\dfrac{3\pi}{4}\right)\right]$ (d) $\cos\left(-\dfrac{\pi}{2}\right) + \mathrm{i}\sin\left(-\dfrac{\pi}{2}\right)$

Working:

(a) $2\sqrt{3}\left[\cos\left(\dfrac{\pi}{3}\right) + \mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right] = \sqrt{3} + 3\mathrm{i}$

(b) $8(\cos 0 + \mathrm{i}\sin 0) = 8$

(c) $2\sqrt{2}\left[\cos\left(-\dfrac{3\pi}{4}\right) + \mathrm{i}\sin\left(-\dfrac{3\pi}{4}\right)\right] = -2 - 2\mathrm{i}$

(d) $\cos\left(-\dfrac{\pi}{2}\right) + \mathrm{i}\sin\left(-\dfrac{\pi}{2}\right) = -\mathrm{i}$

Video: Real and imaginary numbers
Video: Complex number arithmetic
Video: Square rooting complex numbers
Video: Complex conjugates
Video: Division of complex numbers
Video: Argand diagram
Video: Modulus-argument form of complex number
Video: Loci of a circle
Video: Loci of perpendicular bisectors
Video: Loci of half-lines
Video: Representing regions in Argand diagrams
Video: Operations in modulus-argument form

Solutions to Starter and E.g.s

Exercise
No exercise