

Some features of polar coordinates

Starter

1. **(Review of last lesson)** The point A has polar coordinates $\left(5, \frac{\pi}{7}\right)$. Given that OAB form an equilateral triangle, find the Cartesian coordinates of the possible points of B .

Working: B could have polar coordinates $\left(5, \frac{\pi}{7} + \frac{\pi}{3}\right) = \left(5, \frac{10\pi}{21}\right)$

In this case, the Cartesian coordinates of point B are:

$$\left(5 \cos \frac{10\pi}{21}, 5 \sin \frac{10\pi}{21}\right) = (0.3737, 4.986) \text{ (4 s.f.)}$$

B could also have polar coordinates $\left(5, \frac{\pi}{7} - \frac{\pi}{3}\right) = \left(5, -\frac{4\pi}{21}\right) = \left(5, \frac{38\pi}{21}\right)$

In this case, the Cartesian coordinates of point B are:

$$\left(5 \cos \frac{38\pi}{21}, 5 \sin \frac{38\pi}{21}\right) = (4.131, -2.817) \text{ (4 s.f.)}$$

- E.g. 1** Find the maximum and minimum values of r for the curve $r = 8 - 3 \sin 4\theta$.

Working: $-1 \leq \sin 4\theta \leq 1$

The maximum is when $\sin 4\theta = -1$ so maximum is $8 + 3 = 11$.

The minimum is when $\sin 4\theta = 1$ so minimum is $8 - 3 = 5$.

- E.g. 2** Find the maximum and minimum values of r for the curve $r = 7 + 3 \cos^2 \theta$.

Working: $-1 \leq \cos \theta \leq 1 \Rightarrow 0 \leq \cos^2 \theta \leq 1$

The maximum is when $\cos^2 \theta = 1$ so maximum is $7 + 3 = 10$.

The minimum is when $\cos^2 \theta = 0$ so minimum is $7 + 0 = 7$.

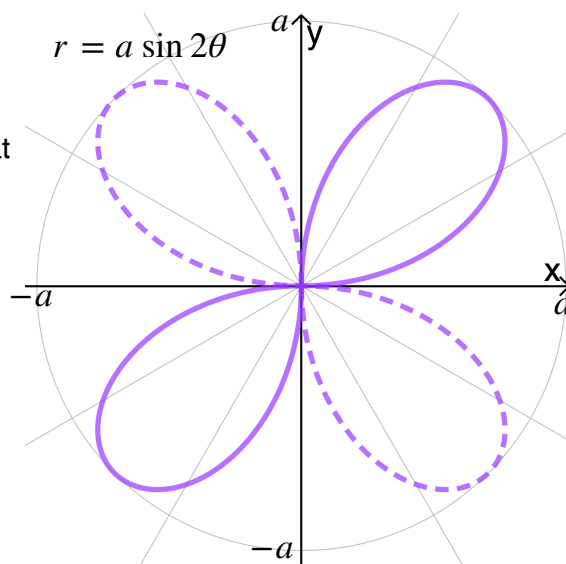
- E.g. 3** State the equations of the other tangents at the pole of $r^2 = a^2 \cos 2\theta$

Working: The other tangents at the pole are $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$.

- E.g. 4** Using the graph of $r = a \cos 2\theta$, state the equations of the tangents at the pole.

Working: The equations of the tangents at the pole are:

$$\begin{aligned} \theta &= 0 \\ \theta &= \frac{\pi}{2}, \\ \theta &= \pi \\ \theta &= \frac{3\pi}{2} \end{aligned}$$



Video: [Sketching polar curves \(spirals\)](#)
Video: [Sketching polar curves \(cardioids\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

p205 9B Qu 1i, 2i, 3-5