The single-sample Wilcoxon signed-rank test

Starter

1. (Review of last lesson)

Carry out single-sample sign tests, at the $5\ \%$ level, on the following data sets against

H_0 : Median is 30

H_1 : Median is more than 30.

State your conclusions clearly.

(a)	26	28	29	32	33	35	36	39	40	41	43	44
(b)	21	23	24	37	38	40	41	44	45	46	48	49

Working: (a) Let X be the number of values above 30 so $X \sim \mathrm{B}(12, 0.5)$

The signs of the differences from the median are:

$- \quad - \quad - \quad + \quad + \quad + \quad + \quad + \quad + \quad + \quad + \quad +$

The sample has 9 values above the median.

$p = P(X \geq 9) = P(X \leq 3) = 0.0730$

Since $p = 0.0730 \nleq 0.05$, we do not reject H_0.

There is evidence to suggest that the median is not more than 30.

(b) Since the signs of the differences are exactly the same as in (a) the working and conclusions are identical.

E.g. 1 State the critical values for the following single-sample Wilcoxon signed-ranks tests:

(a) a one-tailed test at the $1\ \%$ level with 9 values

(b) a one-tailed test at the $5\ \%$ level with 14 values

(c) a two-tailed test at the $5\ \%$ level with 16 values

Working: (a) 3 (b) 25 (c) 29

E.g. 2 A random sample of times, in hours, for cars driving from Perth to Melbourne is given.

$$63.6 \quad 55.2 \quad 54.1 \quad 68.1 \quad 52.3 \quad 47.1 \quad 125.9 \quad 60.8 \quad 61.4 \quad 53.6$$

Assuming that the distribution of times is symmetrical, carry out a single-sample Wilcoxon signed-rank test to test the null hypothesis that the median time for this journey is 55 hours against the alternative hypothesis that the median time for this journey is greater than 55 hours. Use a $5\,\%$ significance level.

Working: H_0 : the population median is 55 hours
H_1 : the population median is greater than 55 hours

Value	63.6	55.2	54.1	68.1	52.3	47.1	125.9	60.8	61.4	53.6
Difference	8.6	0.2	−0.9	13.1	−2.7	−7.9	70.9	5.8	6.4	−1.4
|Difference|	8.6	0.2	0.9	13.1	2.7	7.9	70.9	5.8	6.4	1.4
Rank	8	1	2	9	4	7	10	5	6	3
Signed rank	8	1	−2	9	−4	−7	10	5	6	−3

$W_+ = 39$ and $W_- = 16$

Check: when $n = 10$, $\frac{1}{2} \times 10 \times (10 + 1) = 55 = 39 + 16$ ✓

$T = 16$ (smallest value)

From tables, the critical value for a one-tail test at the $5\,\%$ level with 10 values is 10.

Since $T = 16 \nleq 10 = CV$, we do not reject H_0.

There is evidence to suggest that the population median is 55 hours.

E.g. 3 A bus is timetabled to arrive at $15\ 35$. Its actual arrival time is noted on 10 randomly chosen occasions and the numbers of minutes late or early are recorded, correct to the nearest half-minute. The data is recorded below with positive entries indicating a late time and a negative entry showing an early time.

$$0.5 \quad 3.0 \quad 2.5 \quad -1.0 \quad -3.5 \quad 4.0 \quad 10.0 \quad 5.0 \quad -2.0 \quad 7.5$$

Assuming that the distribution is symmetrical, carry out a single-sample Wilcoxon signed-rank test at the $2.5\ \%$ significance level to test the claim that the bus is late more often than not.

Working: H_0 : the bus scheduled to arrive at $15\ 35$ is not late more often than not
H_1 : the bus scheduled to arrive at $15\ 35$ is late more often than not

Value	0.5	3.0	2.5	−1.0	−3.5	4.0	10.0	5.0	−2.0	7.5
Difference	0.5	3.0	2.5	−1.0	−3.5	4.0	10.0	5.0	−2.0	7.5
|Difference|	0.5	3.0	2.5	1.0	3.5	4.0	10.0	5.0	2.0	7.5
Rank	1	5	4	2	6	7	10	8	3	9
Signed rank	1	5	4	−2	−6	7	10	8	−3	9

$W_+ = 44$ and $W_- = 11$

Check: when $n = 10$, $\frac{1}{2} \times 10 \times (10 + 1) = 55 = 44 + 11$ ✓

$T = 11$ (smallest value)

From tables, the critical value for a one-tail test at the $2.5\ \%$ level with 10 values is 8.

Since $T = 11 \nleq 8 = CV$, we do not reject H_0.

There is evidence to suggest that the bus scheduled to arrive at $15\ 35$ is not late more often than not.

E.g. 4 Becotide inhalers for asthmatics should deliver 50 mg of the active ingredient per puff. In a test, 14 puffs from randomly selected inhalers were tested and the amount of active ingredient delivered was:

$$43.1 \quad 47.3 \quad 52.4 \quad 51.2 \quad 44.7 \quad 50.8$$
$$51.8 \quad 46.7 \quad 52.0 \quad 50.5 \quad 47.7 \quad 45.3$$

(a) Explain why a Wilcoxon test is preferable to a sign test in this context.

(b) Use a single-sample signed-rank test at the $10\,\%$ level to test whether the inhalers are delivering the correct amount of active ingredient.

Working: H_0 : Becotide inhalers deliver 50 mg of the active ingredient per puff.
H_1 : Becotide inhalers do not deliver 50 mg of the active ingredient per puff.

Value	43.1	47.3	52.4	51.2	44.7	50.8	51.8	46.7	48.0	50.5	47.7	45.3
Difference	−6.9	−2.7	2.4	1.2	−5.3	0.8	1.8	−3.3	−2.0	0.5	−2.3	−4.7
|Difference|	6.9	2.7	2.4	1.2	5.3	0.8	1.8	3.3	2.0	0.5	2.3	4.7
Rank	12	8	7	3	11	2	4	9	5	1	6	10
Signed rank	−12	−8	7	3	−11	2	4	−9	−5	1	−6	−10

$W_+ = 17$ and $W_- = 61$

Check: when $n = 12$, $\frac{1}{2} \times 12 \times (12+1) = 22 + 56 = 78$ ✓

$T = 17$ (smallest value)

From tables, the critical value for a two-tail test at the $10\,\%$ level with 12 values is 17.

Since $T = 17 \leq 17 = CV$, we reject H_0.

There is evidence to suggest that the inhalers are not delivering the correct amount of active ingredient.

Video (password needed): Single-sample Wlicoxon signed-rank test
Video: Single-sample Wlicoxon signed-rank test
Video: Wilcoxon signed-rank test

Solutions to Starter and E.g.s

Exercise

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