

The single-sample Wilcoxon signed-rank test

Starter

1. (Review of last lesson)

Carry out single-sample sign tests, at the 5% level, on the following data sets against

H_0 : Median is 30

H_1 : Median is more than 30.

State your conclusions clearly.

(a) 26 28 29 32 33 35 36 39 40 41 43 44

(b) 21 23 24 37 38 40 41 44 45 46 48 49

Working: (a) Let X be the number of values above 30 so $X \sim B(12, 0.5)$

The signs of the differences from the median are:

- - - + + + + + + + +

The sample has 9 values above the median.

$$p = P(X \geq 9) = P(X \leq 3) = 0.0730$$

Since $p = 0.0730 \not\leq 0.05$, we do not reject H_0 .

There is evidence to suggest that the median is not more than 30.

(b) Since the signs of the differences are exactly the same as in (a) the working and conclusions are identical.

E.g. 1 State the critical values for the following single-sample Wilcoxon signed-ranks tests:

(a) a one-tailed test at the 1% level with 9 values

(b) a one-tailed test at the 5% level with 14 values

(c) a two-tailed test at the 5% level with 16 values

Working: (a) 3 (b) 25 (c) 29

E.g. 2 A random sample of times, in hours, for cars driving from Perth to Melbourne is given.

63.6 55.2 54.1 68.1 52.3 47.1 125.9 60.8 61.4 53.6

Assuming that the distribution of times is symmetrical, carry out a single-sample Wilcoxon signed-rank test to test the null hypothesis that the median time for this journey is 55 hours against the alternative hypothesis that the median time for this journey is greater than 55 hours. Use a 5% significance level.

Working: H_0 : the population median is 55 hours
 H_1 : the population median is greater than 55 hours

| | | | | | | | | | | |
|---------------------|------|------|------|------|------|------|-------|------|------|------|
| Value | 63.6 | 55.2 | 54.1 | 68.1 | 52.3 | 47.1 | 125.9 | 60.8 | 61.4 | 53.6 |
| Difference | 8.6 | 0.2 | -0.9 | 13.1 | -2.7 | -7.9 | 70.9 | 5.8 | 6.4 | -1.4 |
| Difference | 8.6 | 0.2 | 0.9 | 13.1 | 2.7 | 7.9 | 70.9 | 5.8 | 6.4 | 1.4 |
| Rank | 8 | 1 | 2 | 9 | 4 | 7 | 10 | 5 | 6 | 3 |
| Signed rank | 8 | 1 | -2 | 9 | -4 | -7 | 10 | 5 | 6 | -3 |

$$W_+ = 39 \text{ and } W_- = 16$$

$$\text{Check: when } n = 10, \frac{1}{2} \times 10 \times (10 + 1) = 55 = 39 + 16 \quad \checkmark$$

$$T = 16 \text{ (smallest value)}$$

From tables, the critical value for a one-tail test at the 5% level with 10 values is 10.

Since $T = 16 \not\leq 10 = CV$, we do not reject H_0 .

There is evidence to suggest that the population median is 55 hours.

E.g. 3 A bus is timetabled to arrive at 15 35. Its actual arrival time is noted on 10 randomly chosen occasions and the numbers of minutes late or early are recorded, correct to the nearest half-minute. The data is recorded below with positive entries indicating a late time and a negative entry showing an early time.

0.5 3.0 2.5 -1.0 -3.5 4.0 10.0 5.0 -2.0 7.5

Assuming that the distribution is symmetrical, carry out a single-sample Wilcoxon signed-rank test at the 2.5 % significance level to test the claim that the bus is late more often than not.

Working: H_0 : the bus scheduled to arrive at 15 35 is not late more often than not
 H_1 : the bus scheduled to arrive at 15 35 is late more often than not

| | | | | | | | | | | |
|---------------------|-----|-----|-----|------|------|-----|------|-----|------|-----|
| Value | 0.5 | 3.0 | 2.5 | -1.0 | -3.5 | 4.0 | 10.0 | 5.0 | -2.0 | 7.5 |
| Difference | 0.5 | 3.0 | 2.5 | -1.0 | -3.5 | 4.0 | 10.0 | 5.0 | -2.0 | 7.5 |
| Difference | 0.5 | 3.0 | 2.5 | 1.0 | 3.5 | 4.0 | 10.0 | 5.0 | 2.0 | 7.5 |
| Rank | 1 | 5 | 4 | 2 | 6 | 7 | 10 | 8 | 3 | 9 |
| Signed rank | 1 | 5 | 4 | -2 | -6 | 7 | 10 | 8 | -3 | 9 |

$$W_+ = 44 \text{ and } W_- = 11$$

$$\text{Check: when } n = 10, \frac{1}{2} \times 10 \times (10 + 1) = 55 = 44 + 11 \quad \checkmark$$

$$T = 11 \text{ (smallest value)}$$

From tables, the critical value for a one-tail test at the 2.5 % level with 10 values is 8.

Since $T = 11 \not\leq 8 = CV$, we do not reject H_0 .

There is evidence to suggest that the bus scheduled to arrive at 15 35 is not late more often than not.

E.g. 4 Becotide inhalers for asthmatics should deliver 50 mg of the active ingredient per puff. In a test, 12 puffs from randomly selected inhalers were tested and the amount of active ingredient delivered was:

43.1 47.3 52.4 51.2 44.7 50.8
51.8 46.7 48.0 50.5 47.7 45.3

- (a) Explain why a Wilcoxon test is preferable to a sign test in this context.
(b) Use a single-sample signed-rank test at the 10% level to test whether the inhalers are delivering the correct amount of active ingredient.

Working: H_0 : Becotide inhalers deliver 50 mg of the active ingredient per puff.
 H_1 : Becotide inhalers do not deliver 50 mg of the active ingredient per puff.

| | | | | | | | | | | | | |
|---------------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Value | 43.1 | 47.3 | 52.4 | 51.2 | 44.7 | 50.8 | 51.8 | 46.7 | 48.0 | 50.5 | 47.7 | 45.3 |
| Difference | -6.9 | -2.7 | 2.4 | 1.2 | -5.3 | 0.8 | 1.8 | -3.3 | -2.0 | 0.5 | -2.3 | -4.7 |
| Difference | 6.9 | 2.7 | 2.4 | 1.2 | 5.3 | 0.8 | 1.8 | 3.3 | 2.0 | 0.5 | 2.3 | 4.7 |
| Rank | 12 | 8 | 7 | 3 | 11 | 2 | 4 | 9 | 5 | 1 | 6 | 10 |
| Signed rank | -12 | -8 | 7 | 3 | -11 | 2 | 4 | -9 | -5 | 1 | -6 | -10 |

$$W_+ = 17 \text{ and } W_- = 61$$

Check: when $n = 12$, $\frac{1}{2} \times 12 \times (12 + 1) = 22 + 56 = 78$ ✓

$$T = 17 \text{ (smallest value)}$$

From tables, the critical value for a two-tail test at the 10% level with 12 values is 17.

Since $T = 17 \leq 17 = CV$, we reject H_0 .

There is evidence to suggest that the inhalers are not delivering the correct amount of active ingredient.

Video (password needed):
Video:

[Single-sample Wilcoxon signed-rank test](#)
[Single-sample Wilcoxon signed-rank test](#)
Video: [Wilcoxon signed-rank test](#)

[Solutions to Starter and E.g.s](#)

Exercise

p51 4B Qu 1i, 2-4, (5 red)