Using standard Maclaurin series

Starter

1. Given that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots$, find the first four non-zero terms in the expansion of $\ln\left(\frac{1}{4} - x\right)$ in ascending powers of x.

Working: $\ln\left(\frac{1}{4} - x\right) \equiv \ln\frac{1}{4}(1-4x) \equiv \ln\frac{1}{4} + \ln(1-4x)$

$$\ln(1-4x) = (-4x) - \frac{(-4x)^2}{2} + \frac{(-4x)^3}{3} - \ldots$$

So $\ln\left(\frac{1}{4} - x\right) = \ln\frac{1}{4} - 4x - 8x^2 - \frac{64x^3}{3} - \ldots$

E.g. 1 Use known Maclaurin series to find the Maclaurin series for $e^{\sin x}$ up to and including the term in x^4.

Working: There are two methods:

Method 1 — substitute the approximation

$$e^{\sin x} = e^{x - \frac{x^3}{3!} + \ldots}$$

Replace x by $x - \frac{x^3}{3!} + \ldots$ in the expansion of e^x:

$$e^x = 1 + x - \frac{x^3}{3!} + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^2}{2!} + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^3}{3!} + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^4}{4!} + \ldots$$

Ignoring powers above x^4:

$$e^x = 1 + x - \frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{6}x^4 + \frac{1}{6}x^3 + \frac{1}{24}x^3 + \ldots$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \ldots$$

Method 2 — separate the powers of e

$$e^{\sin x} = e^{x - \frac{x^3}{3!} + \ldots}$$

$$= e^x \times e^{-\frac{x^3}{3!}}$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\right)\left(1 + \left(-\frac{x^3}{3!}\right) + \ldots\right)$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^3}{6} - \frac{x^4}{6} + \ldots$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \ldots$$

E.g. 2 Use known Maclaurin series to find the Maclaurin series for $\ln(1 + \sin x)$ up to and including the term in x^4.

Working:

$$1 + \sin x = 1 + x - \frac{x^3}{3!} + \ldots$$

$$\ln(1 + \sin x) = \ln\left(1 + x - \frac{x^3}{3!} + \ldots\right)$$

$$= \left(x - \frac{x^3}{3!} + \ldots\right) - \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^2}{2} + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^3}{3} - \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^4}{4} + \ldots$$

$$= x - \frac{x^3}{6} - \frac{x^2}{2} + \frac{x^4}{6} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots$$

E.g. 3 Use the standard expansions to find the first two non-zero terms in the expansion of $x \cos x - \sin x$. Hence find the limit, as $x \to 0$, of $\frac{x \cos x - \sin x}{x^3}$.

Working:

$$x \cos x = x - \frac{x^3}{2!} + \frac{x^5}{4!} - \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$x \cos x - \sin x = x - \frac{x^3}{2!} + \frac{x^5}{4!} - \ldots - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right)$$

$$= -\frac{1}{3}x^3 + \frac{1}{30}x^5$$

$$\lim_{x \to 0} \frac{x \cos x - \sin x}{x^3} = \lim_{x \to 0} \frac{-\frac{1}{3}x^3 + \frac{1}{30}x^5}{x^3} = -\frac{1}{3}$$

Video: Further Maclaurin series

Maclaurin series EQ

Solutions to Starter and E.g.s

Exercise
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