

Vector form of work done, kinetic energy and power

Starter

1. Let $\mathbf{p} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix}$. Find:

- (a) $\mathbf{p} \cdot \mathbf{q}$ (b) $\mathbf{q} \cdot \mathbf{r}$ (c) the angle between \mathbf{p} and \mathbf{r} .

Working: (a) $\mathbf{p} \cdot \mathbf{q} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} = 5 \times 3 + 8 \times 9 + 7 \times 4 = 115$

(b) $\mathbf{q} \cdot \mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix} = 3 \times 6 + 9 \times (-5) + 4 \times (-7) = -55$

(c) Let the angle between \mathbf{p} and \mathbf{r} be θ .

$$|\mathbf{p}| = \sqrt{5^2 + 8^2 + 7^2} = \sqrt{138}$$

$$|\mathbf{r}| = \sqrt{6^2 + (-5)^2 + (-7)^2} = \sqrt{110}$$

$$\mathbf{p} \cdot \mathbf{r} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix} = 5 \times 6 + 8 \times (-5) + 7 \times (-7) = -59$$

$$\cos \theta = \frac{-59}{\sqrt{138} \times \sqrt{110}} \Rightarrow \theta \approx 118.6$$

The angle between \mathbf{p} and \mathbf{r} is 118.6 (1 d.p.)

E.g. 1 Calculate the work done by a force $\mathbf{F} = (3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$ N when it acts over a displacement $\mathbf{s} = (2\mathbf{i} - \mathbf{j} - 5\mathbf{k})$ m.

Working: Work done = $\mathbf{F} \cdot \mathbf{s} = (3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j} - 5\mathbf{k}) = 11$ J

E.g. 2 A particle of mass 5 kg moving with kinetic energy 15 J has an acceleration $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ as it moves through a displacement $\mathbf{s} = 2\mathbf{j} + 5\mathbf{k}$. Find its final kinetic energy.

Working: Initial KE = 15 so $\frac{1}{2} \times 5 \times \mathbf{u} \cdot \mathbf{u} = 15 \Rightarrow \mathbf{u} \cdot \mathbf{u} = 6$

$$\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{s}: \quad \mathbf{v} \cdot \mathbf{v} = 6 + 2(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (2\mathbf{j} + 5\mathbf{k})$$

$$\mathbf{v} \cdot \mathbf{v} = 6 + 2 = 8$$

$$\text{Final KE} = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} = \frac{1}{2} \times 5 \times 8 = 20$$

Its final kinetic energy is 20 J

E.g. 3 A motor boat is sailing with constant velocity $(32\mathbf{i} + 21\mathbf{j})$ m/s. The motor is producing a force $(25\mathbf{i} + 20\mathbf{j})$ N. Find the power at which the motor is working.

Working: Power = $\mathbf{F} \cdot \mathbf{v} = (25\mathbf{i} + 20\mathbf{j}) \cdot (32\mathbf{i} + 21\mathbf{j}) = 800 + 420 = 1220$ W
The motor is working at 1220 W or 1.22 kW

Exercise

p164 6D Qu 1i, 2i, 3i, 4-10

