# Vector form of work done, kinetic energy and power

**Starter**

1. Let $\mathbf{p} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix}$. Find:

(a) $\mathbf{p} \,.\, \mathbf{q}$ (b) $\mathbf{q} \,.\, \mathbf{r}$ (c) the angle between $\mathbf{p}$ and $\mathbf{r}$.

***Working:*** (a) $\mathbf{p} \,.\, \mathbf{q} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix} . \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} = 5 \times 3 + 8 \times 9 + 7 \times 4 = 115$

(b) $\mathbf{q} \,.\, \mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix} . \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix} = 3 \times 6 + 9 \times (-5) + 4 \times (-7) = -55$

(c) Let the angle between $\mathbf{p}$ and $\mathbf{r}$ be $\theta$.

$|\mathbf{p}| = \sqrt{5^2 + 8^2 + 7^2} = \sqrt{138}$

$|\mathbf{r}| = \sqrt{6^2 + (-5)^2 + (-7)^2} = \sqrt{110}$

$\mathbf{p} \,.\, \mathbf{r} = \begin{pmatrix} 5 \\ 8 \\ 7 \end{pmatrix} . \begin{pmatrix} 6 \\ -5 \\ -7 \end{pmatrix} = 5 \times 6 + 8 \times (-5) + 7 \times (-7) = -59$

$\cos\theta = \dfrac{-59}{\sqrt{138} \times \sqrt{110}} \quad \Rightarrow \quad \theta \approx 118.6$

The angle between $\mathbf{p}$ and $\mathbf{r}$ is $118.6$ (1 d.p.)

***E.g. 1*** Calculate the work done by a force $\mathbf{F} = (3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$ N when it acts over a displacement $\mathbf{s} = (2\mathbf{i} - \mathbf{j} - 5\mathbf{k})$ m.

***Working:*** Work done $= \mathbf{F} \,.\, \mathbf{s} = (3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) \,.\, (2\mathbf{i} - \mathbf{j} - 5\mathbf{k}) = 11$ J

***E.g. 2*** A particle of mass $5$ kg moving with kinetic energy $15$ J has an acceleration $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ as it moves through a displacement $\mathbf{s} = 2\mathbf{j} + 5\mathbf{k}$. Find its final kinetic energy.

***Working:*** Initial KE $= 15$ so $\dfrac{1}{2} \times 5 \times \mathbf{u} \,.\, \mathbf{u} = 15 \quad \Rightarrow \quad \mathbf{u} \,.\, \mathbf{u} = 6$

$\mathbf{v} \,.\, \mathbf{v} = \mathbf{u} \,.\, \mathbf{u} + 2\mathbf{a} \,.\, \mathbf{s}$: $\quad \mathbf{v} \,.\, \mathbf{v} = 6 + 2(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \,.\, (2\mathbf{j} + 5\mathbf{k})$

$\mathbf{v} \,.\, \mathbf{v} = 6 + 2 = 8$

Final KE $= \dfrac{1}{2} m\mathbf{v} \,.\, \mathbf{v} = \dfrac{1}{2} \times 5 \times 8 = 20$

Its final kinetic energy is $20$ J

***E.g. 3*** A motor boat is sailing with constant velocity $(32\mathbf{i} + 21\mathbf{j})$ m/s. The motor is producing a force $(25\mathbf{i} + 20\mathbf{j})$ N. Find the power at which the motor is working.

***Working:*** Power $= \mathbf{F} \,.\, \mathbf{v} = (25\mathbf{i} + 20\mathbf{j}) \,.\, (32\mathbf{i} + 21\mathbf{j}) = 800 + 420 = 1220$ W

The motor is working at $1220$ W or $1.22$ kW

Video (password needed): Work done in vector form

Solutions to Starter and E.g.s

**Exercise**
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