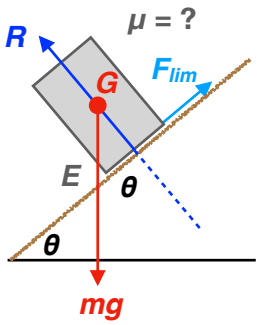
Work done by a variable force

Starter

1. **(Review of last lesson)** A uniform rectangular lamina $ABCD$ is such that $AB = 0.5$ m and $BC = 0.3$ m. The lamina is placed vertically on a rough inclined plane. Find the maximum angle of inclination, and the least coefficient of friction for which the lamina can rest in equilibrium without toppling or sliding, if the side in contact with the plane is
   (a) BC
   (b) AB.

Working: (a) ***Toppling***

When the block is on the point of toppling, the centre of mass is vertically above the edge E.



$$\tan\theta = \frac{0.3}{0.5} \quad \text{so} \quad \theta = 31.0^\circ.$$

Sliding

$R(//)$: $\quad F_{lim} = mg\sin\theta$

$R(\perp)$: $\quad R = mg\cos\theta$

When the block is on the point of sliding $F_{lim} = \mu R$:

$$mg\sin\theta = \mu \times mg\cos\theta$$

$$\mu = \tan\theta = \frac{3}{5}$$

The maximum angle is 31.0° and the least value of the coefficient of friction is $\frac{3}{5}$.

(b) Following similar working to (a), the maximum angle is 59.0° and the least value of the coefficient of friction is $\frac{5}{3}$.

2. An increasing horizontal force P newtons is applied to a block of mass 4 kg whose height is 40 cm and whose length is 70 cm until the block moves. The coefficient between the block and horizontal plane is 0.7. Find the magnitude of P if the block is:
   (a) on the point of sliding
   (b) on the point of toppling.
   (c) Does the block slide or topple?

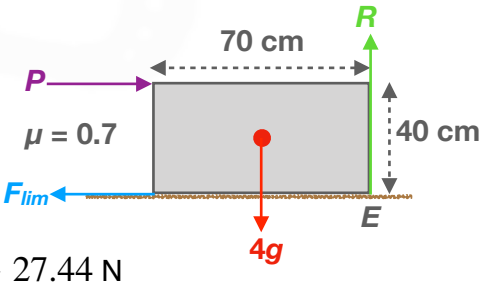
Working: (a) $R(\rightarrow)$: $\quad P = F_{lim}$

$R(\uparrow)$: $\quad R = 4g$

If sliding is about to occur:

$$F_{lim} = \mu R$$
$$= 0.7 \times 4g$$
$$= 27.44 \text{ N}$$

So to make the block slide, $P > 27.44$ N



(b) If the block is about to topple, then the bottom right edge (E) is the pivot point and the reaction of the plane acts at this point.

Moments about E: $\quad P \times 0.4 - 4g \times 0.35 = 0$

so $P = 34.3$ N

So to make the block topple, $P > 34.3$ N

(c) Since $27.44 < 34.3$, the block will slide before it topples.

(Review of AS FM material)

Work done = force × distance

3. A box of mass 6 kg is pulled at a constant speed of 4 m/s for 20 seconds by an inelastic rope along a horizontal surface. Find the work down if the tension in the rope is 18 N.

Working: Distance travelled $= 4 \times 20 = 80$ m
Work done $= 18 \times 80 = 1440$ J

4. Find the work down against gravity if a workman of mass 87 kg climbs a vertical ladder of length 7 m.

Working: Work done $= 87g \times 7 = 5968.2$ J

5. A block of mass 14 kg is pulled 6 m up a plane inclined at 20° to the horizontal. The contact is rough and the magnitude of the frictional force is 30 N. Assuming constant speed, find the work down against:
(a) friction
(b) gravity.

Working: (a) Work done against friction $= 30 \times 6 = 180$ J

(b) Height raised $= 6 \sin 20$
Work done against gravity $= 14g \times 6 \sin 20 = 282$ J (3 s.f.)

E.g. 1 A car of mass 1020 kg moves from rest at A on a horizontal road. The driving force is constant at 1800 N and resistance to motion is modelled as $\frac{x^2}{8}$ N. The car moves 120 m to B.
(a) Find the work down by the driving force and the work done against resistance as the car travels.
(b) Find the speed of the car at B.

Working: (a) WD by driving force $= 1800 \times 120 = 216000$ J
WD against resistance $= \int_0^{120} \frac{x^2}{8} dx = 72000$ J

(b) Increase in KE $= 216000 - 72000 = 144000$ J
So $\frac{1}{2} \times 1020 \times v^2 = 144000$
$\therefore v = \frac{40\sqrt{51}}{17} \approx 16.8$ m/s
The speed of the car at B is 16.8 m/s (3 s.f.).

E.g. 2 An object is moving in a horizontal line against a resistive force that is directly proportional to its distance from its starting point, $f(x) = kx$. If the work done against resistance as the object travels from the origin to a point 15 m away is 337.5 kJ,

(a) find the magnitude of k

(b) state the units of k.

Working: (a) WD against resistance $= \int_0^{15} kx\,dx = \left[\frac{1}{2}kx^2\right]_0^{15} = 112.5k$

WD $= 337500$ J: $112.5k = 337500$

$k = 3000$

(b) $f(x)$ is a force (unit N), and x is a distance (unit m).

Since $k = \frac{f(x)}{x}$, k has units N m^{-1}

Alternatively: replacing N by kg m s^{-2} we get kg s^{-2} .

Video: Work done by a variable force

Video (password needed): Work done by a variable force

Solutions to Starter and E.g.s

Exercise

p141 6A Qu 1-10, (11-12 red)