

Distributions of related continuous random variables [49]

1.

The lifetime of a particular machine, in months, can be modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \frac{3}{t^4} & t \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Obtain the (cumulative) distribution function of T . [2]

(ii) Show that the probability density function of the random variable Y , where $Y = T^3$, is given by $g(y) = \frac{1}{y^2}$, for $y \geq 1$. [6]

(iii) Find $E(\sqrt{Y})$. [3]

[11 marks]

2.

The continuous random variable X has (cumulative) distribution function given by

$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{4}{3} \left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 2, \\ 1 & x > 2. \end{cases}$$

(i) Find the median value of X . [2]

(ii) Find the (cumulative) distribution function of Y , where $Y = \frac{1}{X^2}$, and hence find the probability density function of Y . [6]

(iii) Evaluate $E\left(2 - \frac{2}{X^2}\right)$. [3]

[11 marks]

3.

The object distance U and the image distance V for a convex lens are variables. They are related to the focal length f of the lens, which is a constant, by the formula

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{f}.$$

U is a random variable with a continuous uniform distribution over the interval from $3f$ to $4f$. Find the probability density function of V . [6]

[6 marks]

4.

A rectangle of area $A \text{ m}^2$ has a perimeter of 20m and each of the two shorter sides are of length $X\text{m}$, where X is uniformly distributed between 0 and 2.

(i) Write down an expression for A in terms of X , and hence show that $A = 25 - (X - 5)^2$. [3]

(ii) Write down the probability density function of X . [1]

(iii) Show that the cumulative distribution function of A is

$$F(a) = \begin{cases} 0 & a < 0, \\ \frac{1}{2}(5 - \sqrt{25 - a}) & 0 \leq a \leq 16, \\ 1 & a > 16. \end{cases} \quad [5]$$

(iv) Find the probability density function of A . [2]

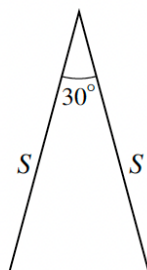
[11 marks]

5.

The continuous random variable S has probability density function given by

$$f(s) = \begin{cases} \frac{8}{3s^3} & 1 \leq s \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

An isosceles triangle has equal sides of length S , and the angle between them is 30° (see diagram).



(i) Find the (cumulative) distribution function of the area X of the triangle, and hence show that the probability density function of X is $\frac{1}{3x^2}$ over an interval to be stated. [7]

(ii) Find the median value of X . [3]

[10 marks]