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A-level  
**MATHEMATICS**  
**7357/2**

Paper 2

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Mark scheme

June 2020

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Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Mark scheme instructions to examiners

### General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

### Key to mark types

|   |  |
|---|--|
| M | mark is for method   |
| R | mark is for reasoning  |
| A | mark is dependent on M or m marks and is for accuracy              |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation  |
| F | follow through from previous incorrect result                      |

### Key to mark scheme abbreviations

|         |   |
|---------|---|
| CAO     | correct answer only   |
| CSO     | correct solution only   |
| ft      | follow through from previous incorrect result                     |
| 'their' | Indicates that credit can be given from previous incorrect result |
| AWFW    | anything which falls within                                       |
| AWRT    | anything which rounds to  |
| ACF     | any correct form  |
| AG      | answer given  |
| SC      | special case  |
| OE      | or equivalent   |
| NMS     | no method shown   |
| PI      | possibly implied  |
| SCA     | substantially correct approach                                    |
| sf      | significant figure(s)   |
| dp      | decimal place(s)  |

**AS/A-level Maths/Further Maths assessment objectives**

| <b>AO</b>  | <b>Description</b>   |
|------------|--|
| <b>AO1</b> | AO1.1a Select routine procedures   |
|            | AO1.1b Correctly carry out routine procedures  |
|            | AO1.2 Accurately recall facts, terminology and definitions                               |
| <b>AO2</b> | AO2.1 Construct rigorous mathematical arguments (including proofs)                       |
|            | AO2.2a Make deductions   |
|            | AO2.2b Make inferences   |
|            | AO2.3 Assess the validity of mathematical arguments                                      |
|            | AO2.4 Explain their reasoning  |
|            | AO2.5 Use mathematical language and notation correctly                                   |
| <b>AO3</b> | AO3.1a Translate problems in mathematical contexts into mathematical processes           |
|            | AO3.1b Translate problems in non-mathematical contexts into mathematical processes       |
|            | AO3.2a Interpret solutions to problems in their original context                         |
|            | AO3.2b Where appropriate, evaluate the accuracy and limitations of solutions to problems |
|            | AO3.3 Translate situations in context into mathematical models                           |
|            | AO3.4 Use mathematical models  |
|            | AO3.5a Evaluate the outcomes of modelling in context                                     |
|            | AO3.5b Recognise the limitations of models   |
|            | AO3.5c Where appropriate, explain how to refine models                                   |

Examiners should consistently apply the following general marking principles

### **No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

### **Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

### **Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

### **Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|------------------------|------|----------|-------------------|
| 1            | Circles correct answer | 2.2a | B1       | $f(x) = -e^{x-1}$ |
| <b>Total</b> |                        |      | <b>1</b> |                   |

| Q            | Marking instructions | AO   | Marks    | Typical solution |
|--------------|----------------------|------|----------|------------------|
| 2            | Ticks correct box    | 2.2a | B1       | $\sec x = 0$     |
| <b>Total</b> |                      |      | <b>1</b> |                  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 3            | Uses the product of $(2x)^5$ and $\left(\pm\frac{3}{x}\right)^3$ terms condone sign error and/or omission of ${}^nC_r$ term<br>Or<br>Obtains any two correct terms (unsimplified) term | 3.1a | M1       | ${}^8C_3(2x)^5 \times \left(-\frac{3}{x}\right)^3 = 56 \times 32 \times -27x^2$ $\therefore \text{coefficient is } -48384$ |
|              | Multiplies their $(2x)^5$ and $\left(-\frac{3}{x}\right)^3$ by ${}^8C_3$ or ${}^8C_5$ or 56 OE<br>For this mark condone $(2x)^3$ and $\left(-\frac{3}{x}\right)^5$                     | 1.1a | M1       |  |
|              | Obtains correct coefficient of $x^2$ –48384<br>condone inclusion $x^2$   | 1.1b | A1       |  |
| <b>Total</b> |  |      | <b>3</b> |  |

| Q            | Marking instructions  | AO   | Marks    | Typical solution   |
|--------------|---|------|----------|--|
| 4            | Uses or states small angle approximation for $\tan 5x \approx 5x$   | 1.1b | B1       | $\frac{x \tan 5x}{\cos 4x - 1} \approx \frac{x \times 5x}{1 - \frac{(4x)^2}{2} - 1}$ $\approx \frac{5x^2}{-8x^2}$ $\approx -\frac{5}{8}$ |
|              | Uses or states small angle approximation for $\cos 4x \approx 1 - \frac{(4x)^2}{2}$<br>Condone omission of bracket  | 1.1b | B1       |  |
|              | Substitutes their expressions<br>Of the form $\tan 5x \approx mx$ and $\cos 4x \approx 1 - \frac{nx^2}{2}$<br>into $\frac{x \tan 5x}{\cos 4x - 1}$<br>Condone correct extra terms | 1.1b | M1       |  |
|              | Deduces $A = -\frac{5}{8}$ from a reasoned argument<br>CSO  | 2.2a | R1       |  |
| <b>Total</b> |   |      | <b>4</b> |  |

| Q | Marking instructions   | AO   | Marks    | Typical solution   |
|---|--|------|----------|--|
| 5 | Uses a suitable substitution<br>$u = 4x + 1$ or $u = \sqrt{4x + 1}$ OE   | 3.1a | M1       | $u = 4x + 1 \Rightarrow \frac{du}{dx} = 4$ $x = -\frac{1}{4} \Rightarrow u = 0$ $x = 6 \Rightarrow u = 25$ $x = \frac{u - 1}{4}$ $\int_{-\frac{1}{4}}^6 x\sqrt{4x + 1} \, dx = \int_0^{25} \frac{u - 1}{4} \sqrt{u} \frac{1}{4} \, du$ $= \frac{1}{16} \int_0^{25} u^{\frac{3}{2}} - u^{\frac{1}{2}} \, du$ $= \frac{1}{16} \left[ \frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} \right]_0^{25}$ $= \frac{1}{8} \left( \frac{25^{\frac{5}{2}}}{5} - \frac{25^{\frac{3}{2}}}{3} \right)$ $= \frac{875}{12}$ |
|   | Differentiates their substitution correctly  | 1.1b | A1       |  |
|   | Completes substitution to obtain correct integrand for their suitable substitution. Can be unsimplified.                                       | 1.1a | M1       |  |
|   | Correctly integrates their simplified integrand provided it is of the form $A\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)$ or $B(u^4 - u^2)$ | 1.1a | A1       |  |
|   | Substitutes correct limits for their substitution or 6 and -1/4 for x  | 1.1a | M1       |  |
|   | Completes rigorous argument to show the required result.<br>AG   | 2.1  | A1       |  |
|   | <b>Total</b>   |      | <b>6</b> |  |



| Q    | Marking instructions   | AO   | Marks    | Typical solution  |
|------|--|------|----------|---|
| 6(a) | Begins a valid method to find the coordinates<br>Uses gradient of $L$ to find gradient of perpendicular radius<br>Or<br>Forms equation of circle with unknown radius and solves simultaneously with equation of $L$<br>Or differentiates equation of circle implicitly   | 3.1a | M1       | $5y + 12x = 298$ $y = \frac{-12}{5}x + \frac{298}{5}$ $y - 9 = \frac{5}{12}(x - 7)$ |
|      | Uses (7, 9) to find the equation of the radius<br>Or<br>Uses (7, 9) correctly in their equation of circle<br>Or<br>Uses $\frac{-12}{5}$ after their implicit differentiation   | 1.1a | M1       | $12y - 5x = 73$ $x = 19$ $y = 14$ $(19, 14)$  |
|      | Obtains $12y - 5x = 73$ OE<br>Or<br>Correctly eliminates a variable to obtain a quadratic in $x$ or $y$ for example<br>obtain a quadratic in $x$ or $y$<br>$(x - 7)^2 + \left(\frac{298 - 12x}{5} - 9\right)^2 = k$ $\Rightarrow (x - 7)^2 + \left(\frac{253 - 12x}{5}\right)^2 = k$ $\left(\frac{298 - 5y}{12} - 7\right)^2 + (y - 9)^2 = k$ $\Rightarrow \left(\frac{214 - 5y}{12}\right)^2 + (y - 9)^2 = k$ | 1.1b | A1       |   |
|      | Equates discriminant to zero<br>PI By correct answer<br>or<br>Solves their simultaneous equations of tangent and radius<br>PI by correct answer  | 3.1a | M1       |   |
|      | Obtains correct values for $x$ and $y$<br>(19, 14)   | 1.1b | A1       |   |
|      | <b>Subtotal</b>  |      | <b>5</b> |   |

|             |  |      |          |  |
|-------------|--|------|----------|--|
| <b>6(b)</b> | Obtains<br>$(x-7)^2 + (y-9)^2 = r^2$<br><b>PI</b> if $r^2$ is replaced with correct value using their point from (a) | 1.1a | M1       | $(x-7)^2 + (y-9)^2 = r^2$<br>$(19-7)^2 + (14-9)^2 = r^2$<br>$12^2 + 5^2 = 169$ |
|             | Uses their point to find radius or radius squared.<br>Must have obtained a point in part (a)                         | 1.1a | M1       | $(x-7)^2 + (y-9)^2 = 169$  |
|             | Obtains correct equation of circle<br>CSO<br>ACF   | 1.1b | A1       |  |
|             | <b>Subtotal</b>  |      | <b>3</b> |  |
|             | <b>Question Total</b>  |      | <b>8</b> |  |

| Q        | Marking instructions   | AO   | Mark     | Typical solution  |
|----------|--|------|----------|---|
| 7(a)(i)  | Identifies the error lies in step 1 without contradiction.   | 2.3  | E1       | Mistake is $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$  |
|          | <b>Subtotal</b>  |      | <b>1</b> |   |
| 7(a)(ii) | Recalls correct addition<br>Accept $\frac{b}{ab} + \frac{a}{ab}$   | 1.1b | M1       | $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$<br>$a + b$ is rational and $ab$ is rational and therefore                              |
|          | Completes rigorous argument to complete proof. Must state that $ab$ is rational (and non-zero) and $a + b$ is rational and conclude that $\frac{1}{a} + \frac{1}{b}$ or $\frac{a+b}{ab}$ is rational                                     | 2.1  | R1       | $\frac{1}{a} + \frac{1}{b}$ is rational.  |
|          | <b>Subtotal</b>  |      | <b>2</b> |   |
| 7(b)     | States assumption to begin proof by contradiction may PI by<br>$\frac{a}{b} - x = \frac{c}{d}$ or $x - \frac{a}{b} = \frac{c}{d}$  | 3.1a | M1       | Assume that the difference between a rational and an irrational number is rational.<br><br>$\frac{a}{b} - x = \frac{c}{d}$          |
|          | Uses language and notation correctly to state initial assumptions:<br>States their $a, b, c$ and $d$ are integers<br><b>and</b><br>$x$ is irrational do not accept the irrational written as a fraction<br>Condone missing $b, d \neq 0$ | 2.5  | A1       | Where $a, b, c$ and $d$ are integers, $b, d \neq 0$ and $x$ is irrational   |
|          | Demonstrates that $x$ can be expressed as a rational number by obtaining $x = \frac{ad - cb}{bd}$ OE   | 1.1b | M1       | $x = \frac{a}{b} - \frac{c}{d}$<br>$= \frac{ad}{bd} - \frac{cb}{bd}$<br>$= \frac{ad - cb}{bd}$                                      |
|          | Completes rigorous argument to prove the required result, clearly explaining where the contradiction lies with <b>ALL</b> assumptions correct at the start (including $b, d \neq 0$ )  | 2.1  | R1       | Hence $x$ is rational. This is a contradiction hence the difference of any rational number and any irrational number is irrational. |
|          | <b>Subtotal</b>  |      | <b>4</b> |   |
|          | <b>Question Total</b>  |      | <b>7</b> |   |

| Q        | Marking instructions  | AO   | Mark     | Typical solution  |
|----------|---|------|----------|---|
| 8(a)     | Eliminates t  | 1.1a | M1       | $\frac{y}{2} = t, \quad x = \frac{y^2}{4}$<br>$y^2 = 4x$  |
|          | Writes the Cartesian equation in the required form  | 1.1b | A1       |   |
|          | <b>Subtotal</b>   |      | <b>2</b> |   |
| 8(b)(i)  | Differentiates both $\frac{dx}{dt} = 2t$ ,<br>$\frac{dy}{dt} = 2$ with at least one correct<br>Or<br>Differentiates their $y^2 = 4x$ to<br>obtain $y \frac{dy}{dx} = 4$<br>Or rearranges and differentiates<br>$y = 2\sqrt{x}$ and obtains<br>$\frac{dy}{dx} = Ax^{-\frac{1}{2}}$<br>OE | 3.1a | M1       | $\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 2$<br>$\frac{dy}{dx} = \frac{2}{2a} = \frac{1}{a}$<br>The gradient of a line is equal to<br>the tangent of the angle between<br>the line and the horizontal hence<br>$\tan \theta = \frac{1}{a}$ |
|          | Obtains correct $\frac{dy}{dx}$ at $t = a$  | 1.1b | A1       |   |
|          | Explains that the gradient of a<br>line is the tangent of the angle<br>between the line and the<br>horizontal<br>or<br>shows on right-angled triangle<br>on diagram and links to $\tan \theta$<br><b>and</b><br>concludes $\tan \theta = \frac{1}{a}$                                   | 2.4  | E1       |   |
|          | <b>Subtotal</b>   |      | <b>3</b> |   |
| 8(b)(ii) | Uses formula for gradient of<br>straight line with points A and B<br>Must have $a^2 - 1$ or $1 - a^2$ in<br>denominator   | 1.1a | M1       | $\tan \phi = \frac{2a - 0}{a^2 - 1}$<br>$= \frac{2a}{a^2 - 1}$  |
|          | Obtains correct $\tan \phi$<br>OE   | 1.1b | A1       |   |
|          | <b>Subtotal</b>   |      | <b>2</b> |   |

|                  |  |      |           |   |
|------------------|--|------|-----------|---|
| <b>8(b)(iii)</b> | States double angle formula for $\tan 2\theta$   | 1.2  | B1        | $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ $= \frac{2 \times \frac{1}{a}}{1 - \left(\frac{1}{a}\right)^2}$ $= \frac{2a}{a^2 - 1}$ $= \tan \phi$ |
|                  | Substitutes $\tan \theta = \frac{1}{a}$ into their<br>$\tan 2\theta = \frac{2 \tan \theta}{1 \pm \tan^2 \theta}$ | 1.1a | M1        |   |
|                  | Simplifies and completes argument to show required result  | 2.1  | R1        |   |
|                  | <b>Subtotal</b>  |      | <b>3</b>  |   |
|                  | <b>Question Total</b>  |      | <b>10</b> |   |

| Q               | Marking instructions  | AO   | Marks    | Typical solution   |
|-----------------|---|------|----------|--|
| 9(a)            | Identifies and clearly defines variable for radius of cylinder. Can be shown on diagram or can be implied by use in $V = \pi r^2 h$   | 2.5  | B1       | Radius of cylinder = $r$<br>$h^2 + r^2 = R^2$<br>$V = \pi r^2 h$   |
|                 | Uses Pythagoras to connect $h$ , $r$ and $R$  | 3.1a | M1       |  |
|                 | Eliminates the radius variable to form an expression for the volume of the cylinder in terms of $h$ , completing argument to show given result. Condone undefined $r$   | 2.1  | R1       | $V = \pi(R^2 - h^2)h$<br>$= \pi R^2 h - \pi h^3$   |
| <b>Subtotal</b> |   |      | <b>3</b> |  |
| 9(b)            | Differentiates the expression for volume w.r.t. $h$ with at least one term correct.   | 3.1a | M1       | $\frac{dV}{dh} = \pi R^2 - 3\pi h^2$   |
|                 | Obtains correct $\frac{dV}{dh}$   | 1.1b | A1       | For maximum volume $\frac{dV}{dh} = 0$   |
|                 | Explains that their derivative w.r.t. $h$ equals zero for a maximum or stationary point   | 2.4  | E1       | $\Rightarrow R^2 - 3h^2 = 0$   |
|                 | Equates volume derivative w.r.t. $h$ to zero and correctly obtains a value for $h$ in terms of $R$  | 1.1a | M1       | $h^2 = \frac{R^2}{3} \Rightarrow h = \frac{R}{\sqrt{3}}$   |
|                 | Substitutes their $h$ , in terms of $R$ , from derivative w.r.t. $h$ into volume formula.   | 1.1a | M1       | Hence volume   |
|                 | Obtains the correct max volume in the form $kR^3 - pR^3$ or better  | 3.2a | A1       | $V = \pi R^2 \frac{R}{\sqrt{3}} - \pi \left( \frac{R}{\sqrt{3}} \right)^3$<br>$= \frac{2\sqrt{3}\pi R^3}{9}$ |
|                 | Justifies correct volume in the form $kR^3 - pR^3$ or better form is the maximum<br>eg:<br><ul style="list-style-type: none"><li>• <math>V = 0</math> when <math>h=0</math> or <math>R</math> and <math>V&gt;0</math> in between.</li><li>• Sketches shape of graph passing through the origin with (min on negative side) and max on positive side</li><li>• Obtains <math>\frac{d^2V}{dh^2} = -6\pi h &lt; 0</math></li></ul> NB R1 can be awarded even if E1 is not awarded. | 2.1  | R1       | $\frac{d^2V}{dh^2} = -6\pi h$<br>When $h = \frac{R}{\sqrt{3}}$<br>$\frac{d^2V}{dh^2} < 0$ Therefore maximum  |

|  |                       |  |           |  |
|--|-----------------------|--|-----------|--|
|  | <b>Subtotal</b>       |  | <b>7</b>  |  |
|  | <b>Question Total</b> |  | <b>10</b> |  |

| Q  | Marking Instructions | AO  | Marks    | Typical Solution                                  |
|----|----------------------|-----|----------|---|
| 10 | Ticks correct box    | 1.2 | B1       | The resultant force acting on the vehicle is zero |
|    | <b>Total</b>         |     | <b>1</b> |   |

| Q  | Marking Instructions   | AO   | Marks    | Typical Solution |
|----|------------------------|------|----------|------------------|
| 11 | Circles correct answer | 1.1b | B1       | $\binom{-2}{2}N$ |
|    | <b>Total</b>           |      | <b>1</b> |                  |

| Q  | Marking Instructions   | AO   | Marks    | Typical Solution |
|----|------------------------|------|----------|------------------|
| 12 | Circles correct answer | 1.1b | B1       | -6               |
|    | <b>Total</b>           |      | <b>1</b> |                  |

| Q     | Marking Instructions   | AO   | Marks    | Typical Solution                                    |
|-------|--|------|----------|---|
| 13(a) | Finds a moment of a force about any point.<br>Must have the form force x distance.<br>Can be awarded for example for 2w, 7w, 5R, 2R<br>Condone use of 9.8, 9.81, 10 instead of g | 3.4  | M1       | $2W = 1.5 \times 4g$ $W = 3g$                       |
|       | Obtains $w = 3g$<br>CAO  | 1.1b | A1       |   |
|       | <b>Subtotal</b>  |      | <b>2</b> |   |
| 13(b) | Explains that the weight of the rod acts through the midpoint or the centre of mass is at the midpoint (of the rod) OE   | 3.5a | E1       | The rod is uniform so its weight acts at the centre |
|       | <b>Subtotal</b>  |      | <b>1</b> |   |
|       | <b>Question Total</b>  |      | <b>3</b> |   |

| Q     | Marking Instructions  | AO   | Marks    | Typical Solution  |
|-------|---|------|----------|---|
| 14(a) | Differentiates to find expression for $\mathbf{v}$ with at least one component correct  | 3.4  | M1       | $\mathbf{v} = (3t^2 - 10t)\mathbf{i} + (8 - 2t)\mathbf{j}$ <p>So when <math>t = 2</math></p> $\mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$ $\text{Speed} =  \mathbf{v}  = \sqrt{8^2 + 4^2}$ $= 4\sqrt{5} \text{ m s}^{-1}$  |
|       | Substitutes $t = 2$ into their $\mathbf{v}$ which has at least one component correct.   | 1.1a | M1       |   |
|       | Uses correct process to find the magnitude of their expression for $\mathbf{v}$ provided both of their components are non-zero  | 1.1a | M1       |   |
|       | Obtains correct answer $4\sqrt{5}$<br>ISW<br>ACF<br>CSO   | 1.1b | A1       |   |
|       | <b>Subtotal</b>   |      | <b>4</b> |   |
| 14(b) | Differentiates <i>their</i> vector $\mathbf{v}$ to find expression for $\mathbf{a}$ with at least one component correct.<br>Do not award for $\mathbf{a} = \mathbf{r}$  | 3.4  | M1       | $\mathbf{a} = (6t - 10)\mathbf{i} - 2\mathbf{j}$ $ \mathbf{a}  = \sqrt{(6t - 10)^2 + 2^2}$ <p>Since <math> \mathbf{a}  \geq 2</math> for any value of <math>t</math> the magnitude of the acceleration is never zero</p> <p>Therefore Bella's claim is correct.</p> |
|       | Uses a valid explanation as to why the magnitude must be positive for all values of $t$<br>This could be earned by:<br>forming a quadratic from acceleration components, allow sign error<br>or<br>Stating that the $\mathbf{j}$ component is always $-2$ | 2.4  | E1       |   |
|       | Deduces that $ \mathbf{a}  \geq 2$ OE or that Bella is correct or $ \mathbf{a}  > 0$ .<br>Deduction must be from completely correct reasoning.  | 2.2a | R1       |   |
|       | <b>Subtotal</b>   |      | <b>3</b> |   |
|       | <b>Question Total</b>   |      | <b>7</b> |   |



| Q     | Marking Instructions   | AO   | Marks    | Typical Solution   |
|-------|--|------|----------|--|
| 15(a) | States strip width $h = 20$<br>PI by correct $y$ values and not contradicted.  | 1.1b | B1       | $h = 20$ $y_0 = 131, y_1 = 140,$ $y_2 = 120, y_3 = 80, y_4 = 0$ $\text{Area} = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)$ $\text{Distance} = 8110 \text{ m}$ |
|       | States five $y$ values PI<br>$y_0 = 131, y_1 = 140,$<br>$y_2 = 120, y_3 = 80, y_4 = 0$<br><br>If they use $h=25$ condone<br>$y_0 = 6, y_1 = 135, y_2 = 134,$<br>$y_3 = 94, y_4 = 0$<br>If they use five strips between<br>$20 < t < 100$ condone<br>$y_0 = 131, y_1 = 140, y_2 = 132,$<br>$y_3 = 108, y_4 = 67, y_5 = 0$<br><br>Accept $\pm 2$ on all $y$ values | 1.1a | M1       |  |
|       | Applies correct trapezium rule formula to their values (this mark could be achieved with B0M0 so far)  | 1.1a | M1       |  |
|       | Applies trapezium rule with four strips and obtains correct value for distance. (accept values between 7970m and less than 8250m)  | 1.1a | A1       |  |
|       | <b>Subtotal</b>  |      |          |  |
| 15(b) | Explains means of gaining a more accurate estimate<br>For example, “integrate the quadratic between 20 and 100”<br>Must include limits for integration.<br>Or<br>Use the quadratic to calculate $y$ values in the trapezium rule or other appropriate numerical method.  | 2.4  | E1       | Integrates the quadratic between the limits 20 and 100.  |
|       | <b>Subtotal</b>  |      | <b>1</b> |  |
|       | <b>Question Total</b>  |      | <b>5</b> |  |

| Q  | Marking Instructions   | AO   | Marks    | Typical Solution   |
|----|--|------|----------|--|
| 16 | Uses $s = ut + \frac{1}{2}at^2$ for particle A<br>With $u=0$ $t=5$ and $a=g$<br>( $g$ can be 9.8 9.81, or 10)  | 3.1b | M1       | $s = ut + \frac{1}{2}at^2$<br>$s = h, a = g, u = 0 \text{ and } t = 5$ |
|    | Finds $h$ in terms of $g$<br>Condone use of $g$ , 9.8, 9.81, or 10<br>Can achieve this mark for<br>$h = 12.5g, 122.5, 122.625, 125$                              | 1.1b | A1       | $h = \frac{25}{2}g$<br>$kh = \frac{1}{2}g(5 - t)^2$                    |
|    | Uses $kh = ut + \frac{1}{2}at^2$ for particle B<br>With $u=0$ and $a=g$<br>( $g$ can be 9.8 9.81, or 10)   | 1.1a | M1       | $\frac{25}{2}gk = \frac{1}{2}g(5 - t)^2$                               |
|    | Deduces “time for B” + $t = 5$   | 3.3  | B1       | $5\sqrt{k} = 5 - t, \text{ since } 0 < t < 5$                          |
|    | Eliminates $h$ and completes<br>reasoned argument to obtain<br>$t = 5(1 - \sqrt{k})$<br>with consistent use of $g$ or value for<br>$g$ and no slips<br><b>AG</b> | 2.1  | R1       | $t = 5(1 - \sqrt{k})$  |
|    | <b>Total</b>   |      | <b>5</b> |  |

| Q  | Marking Instructions   | AO   | Marks    | Typical Solution  |
|----|--|------|----------|---|
| 17 | Models vertical motion using a suitable constant acceleration equation, to find the time of flight. Condone consistent sine/ cosine error                | 3.3  | M1       | Vertically:<br>$0 = t \times u \sin \theta - 0.5 \times g \times t^2$                             |
|    | Obtains correct equation or inequality for t<br>Must use g not numerical value   | 1.1b | A1       | $t = \frac{2u \sin \theta}{g}$  |
|    | Models horizontal displacement<br>Condone consistent sine/ cosine error  | 3.3  | M1       | Horizontally:<br>$x = u t \cos \theta$  |
|    | Obtains correct equation or inequality to model horizontal displacement  | 1.1b | A1       | $x = u \frac{2u \sin \theta}{g} \cos \theta,$<br>using $\sin 2\theta = 2 \sin \theta \cos \theta$ |
|    | Eliminates $t$ , where $t$ is time of flight, from their horizontal and vertical models to obtain an expression in terms of $u$ , $g$ and $\theta$ only. | 3.4  | M1       | $x = \frac{u^2 \sin 2\theta}{g}$  |
|    | Completes reasoned argument to obtain stated result. Must justify stated inequality must have used correct resolution of $u$ throughout                  | 2.1  | R1       | Since $x \geq d$<br>$\frac{u^2 \sin 2\theta}{g} \geq d$<br>$\sin 2\theta \geq \frac{dg}{u^2}$     |
|    | <b>Total</b>   |      | <b>6</b> |   |

| Q                     | Marking Instructions   | AO   | Marks     | Typical Solution  |
|-----------------------|--|------|-----------|---|
| 18(a)                 | Uses $F = ma$ to form a four-term equation with at least three correct terms<br>condone consistent swapping of sine/cosine   | 3.3  | M1        | $T - W \sin \theta - \text{Friction} = ma$ $T - mg \sin \theta - \mu mg \cos \theta = ma$ $T - 0.2g(0.28) - 0.2\mu g(0.96)$ $= 0.2 \times \frac{543}{625}g$ $2g - T = 2 \times \frac{543}{625}g$ $2g - 0.2g(0.28) - 0.2\mu g(0.96)$ $= 2.2 \times \frac{543}{625}g$ $0.192\mu = 2 - 0.056 - 1.911(36)$ $\mu = 0.17$ |
|                       | Obtains perpendicular component of weight  | 3.1b | B1        |   |
|                       | Obtains parallel component of weight   | 3.1b | B1        |   |
|                       | Recalls $F = \mu R$  | 1.2  | B1        |   |
|                       | Forms fully correct equation for particle A<br>$T - mg \sin \theta - \mu mg \cos \theta = ma$<br>Obtains M1, B1, B1, B1, A1  | 1.1b | A1        |   |
|                       | Forms fully correct equation for particle B<br>$2g - T = 2a$ or with acceleration substituted  | 3.3  | B1        |   |
|                       | Eliminates $T$ from their equations of motion with $a \neq 0$  | 3.4  | M1        |   |
|                       | Shows at least one step leading to given $\mu$ value AWRT 0.170<br>If exact values used throughout with $g=9.81$ then $\mu = 0.17$ exactly   | 1.1b | A1        |   |
| <b>Subtotal</b>       |  |      | <b>8</b>  |   |
| 18(b)(i)              | Uses $F = ma$ with $T=0$ to form a three-term equation with at least two correct terms<br>condone consistent sine/cosine error condone one sign error                                | 3.4  | M1        | $-0.2 \times 9.81 \times (0.28)$ $- 0.2(0.17) \times 9.81$ $\times (0.96) = 0.2a$ $a = -4.347792$ <p>Use <math>v^2 = u^2 + 2as</math></p> $0 = 0.25 - 8.696s$ $s = 0.0288 \text{ m}$  |
|                       | Obtains $a = -4.347792$<br>AWRT -4.35  | 1.1b | A1        |   |
|                       | Uses $v^2 = u^2 + 2as$ with $u=0.5$ and $v=0$ and their $a$ value<br>Do not accept the following values for $a$<br>$a \neq 0, \frac{543}{625}g, 8.51424$<br>$a \neq 8.522928, 8.688$ | 3.4  | M1        |   |
|                       | Obtains 0.0288 m<br>Must have units<br>CAO   | 3.2a | A1        |   |
| <b>Subtotal</b>       |  |      | <b>4</b>  |   |
| 18(b)(ii)             | Describes assumption in context of part (b)(i) eg No air resistance, String does not obstruct block  | 3.5b | E1        | $A$ does not reach the pulley before coming to rest   |
| <b>Subtotal</b>       |  |      | <b>1</b>  |   |
| <b>Question Total</b> |  |      | <b>13</b> |   |

| Q     | Marking Instructions   | AO   | Marks    | Typical Solution   |
|-------|--|------|----------|--|
| 19(a) | Forms differential equation<br>PI by correctly separated variables<br>or $\int -\frac{1}{v^2} dv = \int 0.1 dt$ OE | 3.4  | B1       | $\frac{dv}{dt} = -0.1v^2$ $\int -\frac{1}{v^2} dv = \int 0.1 dt$ $\frac{1}{v} = 0.1t + c$ $\frac{1}{4} = 0.1 \times (0) + c$ $\frac{1}{v} = 0.1t + 0.25$ $v = \frac{1}{0.1t + 0.25}$ $v = \frac{20}{5 + 2t}$ |
|       | Separates variables to integrate<br>Condone sign error   | 1.1a | M1       |  |
|       | Integrates with one side correct   | 1.1b | A1       |  |
|       | Fully correct integration<br>Condone missing constant of<br>integration  | 1.1b | A1       |  |
|       | Uses initial conditions to correctly<br>obtain an equation in v and t only.  | 3.4  | M1       |  |
|       | Completes rigorous argument to<br>show given expression<br><b>AG</b>   | 2.1  | R1       |  |
|       | <b>Subtotal</b>  |      | <b>6</b> |  |
| 19(b) | Substitutes $t = 5.5$ in $v = \frac{20}{5+2t}$   | 3.4  | M1       | $v = \frac{20}{5 + 11} = 1.25$ $a = \frac{-5}{32} = -0.15625$ Acceleration = $-0.15625 \text{ m s}^{-2}$   |
|       | Obtains correct acceleration<br>$= -0.15625 \text{ m s}^{-2}$<br><br>AWRT - 0.156                                  | 1.1b | A1       |  |
|       | <b>Subtotal</b>  |      | <b>2</b> |  |
|       | <b>Question Total</b>  |      | <b>8</b> |  |