Pearson Edexcel

Mark Scheme (Results)

November 2021

Pearson Edexcel GCE Mathematics

Pure 1 Paper 9MA0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\dots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\dots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

±

Question	Scheme	Marks	AOs
1	$\mathrm{f}(1)=a(1)^3+10(1)^2-3a(1)-4=0$	M1	3.1a
	$6-2a=0 \Rightarrow a=...$	M1	1.1b
	$a=3$	A1	1.1b
		(3)	
(3 marks)			
Notes			

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a+10-3a-4=0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

....

Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a=3,\ \mathrm{f}(x)=3x^3+10x^2-9x-4$ and shows that $(x-1)$ is a factor of this $\mathrm{f}(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3+10x^2-9x-4=(x-1)(3x^2+13x+4)$ Hence $a=3$

E.g. 2: $\mathrm{f}(x)=3x^3+10x^2-9x-4,\quad \mathrm{f}(1)=3+10-9-4=0$ Hence $a=3$

The solutions via this method must end with the value for a to score the A1

....

....
Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

Alt (1) by inspection which may be seen in a table/g

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$ and sets terms in x equal

$-3a=-(10+a)+4 \Rightarrow 2a=6 \Rightarrow a=3$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x
....

Alt (2) By division:

$$\begin{array}{r l}
 & ax^2+(\pm 10\pm a)x+(10-2a) \\
x-1\,\big) & ax^3+10x^2\quad -3ax-4 \\
 & \underline{ax^3-ax^2} \\
 & (10+a)x^2-3ax \\
 & \underline{(10+a)x^2-(10+a)x} \\
 & (-2a+10)x
\end{array}$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm 10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

Question	Scheme	Marks	AOs
2(a)	$\mathrm{f}(x)=(x-2)^2 \pm ...$	M1	1.2
	$\mathrm{f}(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0, 5)$	B1	1.1b
(b)(ii)	$Q=(2, 1)$	B1ft	1.1b
		(2)	
			(4 marks)
	Notes		

(a)

M1: Achieves $(x-2)^2 \pm ...$ or states $a = -2$

A1: Correct expression $(x-2)^2+1$ ISW after sight of this

Condone $a = -2$ and $b = 1$. Condone $(x-2)^2+1=0$

(b)

(i) B1: Correct coordinates for P. Allow to be expressed $x=0,\ y=5$

(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x=2,\ y=1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)

Score in any order if they state $P=(0, 5)$ and $Q=(2, 1)$

....

Allow part (b) to be awarded from a sketch. So award

First B1 from a sketch crossing the y-axis at 5

Second B1 from a sketch with minimum at $(2, 1)$

....

Question	Scheme	Marks	AOs
3(a)	$u_2 = k-12,\ u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0*$	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	k = 6 as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
			(6 marks)
	Notes		

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k - \frac{24}{2}$ o.e. but could be awarded for $u_3 = k - \frac{24}{their\ "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \frac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of $k = 6$.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by $k =$
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least $k = 6$

A1: Chooses k = 6 and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\frac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

Question	Scheme	Marks	AOs
4(a)	$\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$ *	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x) = 2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415) = 0.00366...$ $\mathrm{h}(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)
	Notes		

(a)

M1: Differentiates $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{\mathrm{g}(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $\mathrm{f}'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{\mathrm{g}(x)}{2x^2-4x+5}$ = 0 and uses "**correct''** algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm \mathrm{g}(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1 = 0.3$. If no method is shown award for $x_2 = \text{awrt } 0.33$

A1: $x_2 = \text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4 = \text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2,\ x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.0018

There must be sufficient evidence for the function, which would be for example, a statement such as $\mathrm{h}(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $\mathrm{h}(x)$ being mislabelled as f

$\mathrm{h}(0.3415)=2\times 0.3415^3-4\times 0.3415^2+7\times 0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20\,000 \times 1.08^2 = (£)\,23328$*	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
Notes			

(a)
B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units
E.g. $£20\,000 \times 1.08^2$ or $£20\,000 \times 108\% \times 108\%$
This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.
Condone calculations seen as 8% of 20000 = 1600.
This is a show that question and the method must be seen.
It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example N or n for $n - 1$. So award for $20\,000 \times 1.08^{n-1} > 65\,000$,

$20\,000 \times 1.08^{n} = 65\,000$ or $20\,000 \times (108\%)^{n} \geqslant 65\,000$ amongst others.

Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of $n - 1$, N, n etc.

Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000 \times 1.08^{n} = 65\,000 \Rightarrow n \log 1.08 = \log \dfrac{65000}{20000} \Rightarrow n = \ldots$

E.g. $2000 \times 1.8^{n} = 65\,000 \Rightarrow \log 2000 + n \log 1.8 = \log 65000 \Rightarrow n = \ldots$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n = 16) \Rightarrow P = 20\,000 \times 1.08^{15}$ =awrt 63400 **or** $(n = 17) \Rightarrow P = 20\,000 \times 1.08^{16}$ = awrt 68500

M1: $(n = 16) \Rightarrow P = 20\,000 \times 1.08^{15}$ =awrt 63400 **and** $(n = 17) \Rightarrow P = 20\,000 \times 1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n = 1.08 \times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\frac{50+18-14}{2\sqrt{50}\sqrt{18}}=\frac{9}{10}*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\frac{27}{\sqrt{50}\sqrt{18}}=\frac{9}{10}*$	A1*	2.1
			(5 marks)

Notes

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\frac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\frac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\frac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,\,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k\Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0\Rightarrow(6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
		(9 marks)	

Notes

(a)(i)

M1: Attempts to complete the square on by halving both *x* and *y* terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2 . For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0\Rightarrow(x-a)^2+(y-b)^2=k\Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,\,-2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct *a*, *b* and *c*

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both *b* and *c* are expressions in *k* . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in *k*. See General Principles.

The 3TQ in *k* must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

<u>**Look carefully at the method used. It is possible to attempt this using gradients**</u>

(b) Alt 1	$x^2+y^2-10x+4y+11=0\Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3\Rightarrow x+3y+1=0$ and combines with equation for *C* $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a

	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.

M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0\Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3\ \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k,\ x+3y=1$ $\Rightarrow x=...$ and $y=...$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10},\ y=\frac{k-3}{10},\ x^2+y^2-10x+4y+11=0\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y=3x+k\Rightarrow m=3\Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow\frac{10}{9}(x-5)^2=18\Rightarrow x=...$ or $\Rightarrow10(y+2)^2=18\Rightarrow y=...$	M1
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x\Rightarrow k=...$	M1
	$k=-17\pm6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

M1: Solves simultaneously to find x or y

Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values

Question	Scheme	Marks	AOs
8(a)	$A=1000$	B1	3.4
	$2000=1000\mathrm{e}^{5k}$ or $\mathrm{e}^{5k}=2$	M1	1.1b
	$\mathrm{e}^{5k}=2\Rightarrow 5k=\ln 2\Rightarrow k=...$	M1	2.1
	$N=1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $N=1000\mathrm{e}^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{\mathrm{d}N}{\mathrm{d}t}=1000\times\left(\frac{1}{5}\ln 2\right)\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{\mathrm{d}N}{\mathrm{d}t}=1000\times 0.139\mathrm{e}^{0.139t}$ $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8}=1000\times\left(\frac{1}{5}\ln 2\right)\mathrm{e}^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8}=1000\times 0.139\mathrm{e}^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500\mathrm{e}^{1.4\times\left(\frac{1}{5}\ln 2\right)T}=1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)T}$ or $500\mathrm{e}^{1.4\times\text{"}0.139\text{"}t}=1000\mathrm{e}^{\text{"}0.139\text{"}t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2=\ln 2$ or $1.4\times\text{"}0.339\text{"}T=\ln 2+\text{"}0.339\text{"}t$	M1	2.1
	$T=12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N=1000\mathrm{e}^{...t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N=2000, t=5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $a\mathrm{e}^{5k}=b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N=1000\mathrm{e}^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N=1000\times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates $\alpha\mathrm{e}^{kt}$ to $\beta\mathrm{e}^{kt}$ and substitutes t = 8 (Condone $\alpha=\beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

....

Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000e^{0.139t}$, and then writes at $t = 8$ $\frac{dN}{dt} =$ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.
(c) The first M1 should be seen E.g $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$
If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

....

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B = ...$ or $C = ...$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0$ $x=0 \Rightarrow 9 = 2A+B+4C$ $\Rightarrow 9 = 2A+1+8 \Rightarrow A = ...$	M1	2.1
	$A = 0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + ...$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2} = 1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2 + ...$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2} = \frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2 + ...$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1+2x+\frac{-1(-1-1)}{2!}(2x)^2 + .$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x} = \frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+...+2+4x+8x^2+...$	dM1	2.1
	$= \frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+...$	A1	1.1b
(b)(ii)	$\lvert x\rvert < \frac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)

Notes

(a)(i)
M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x = \frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).
A1: Both values correct
(a)(ii)
M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.
Amongst many different methods are:
Compare terms in x^2 $\Rightarrow 50 = -10A+25C$ which would be implied by $50 = -10A+25\times"2"$

Compare constant terms or substitute $x=0 \Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$
A1*: Fully correct proof with no errors.
Note: The second part is a proof so it is important that a suitable proof/show that is seen.
Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0

(b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2}=2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2}+\ldots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1+(-2)*x+\frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....
Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2}=2^{-2}+(-2)2^{-3}\times 5x+\frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1)*x+\frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.
There must be some attempt to use their values of *B* and *C*
A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5}<x<\frac{2}{5}$ or $x\in\left(-\frac{2}{5},\frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Question	Scheme	Marks	AOs
10(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta\left(\sin\theta+\cos\theta\right)}{2\cos\theta\left(\cos\theta+\sin\theta\right)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x\left(1-3\cos 2x\right)=0$ $\Rightarrow\left(\sin 2x=0,\right)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of ($\sin\theta + \cos\theta$)

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta\cancel{\left(\sin\theta+\cos\theta\right)}}{2\cos\theta\cancel{\left(\cos\theta+\sin\theta\right)}}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \dfrac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \dfrac{1}{3}$ or $\cos^2 x = \dfrac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°, 35°, 145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

....

Answers without working in (b): Just answers and no working score 0 marks.

If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2}\times\frac{1}{2}\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int(\ln x)^2\ dx = x(\ln x)^2 - \int x\times\frac{2\ln x}{x}\ dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x\ dx = x(\ln x)^2 - 2\left(x\ln x - \int dx\right)$ $= x(\ln x)^2 - 2\int \ln x\ dx = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2\ dx = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times 4\ln 4 + 2\times 4 - \left(2(\ln 2)^2 - 2\times 2\ln 2 + 2\times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
			(8 marks)

Notes

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times "h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta\int \ln x\ dx$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x\ dx = x\ln x - x$

who may write $\int(\ln x)^2\ dx = \int(\ln x)(\ln x)\ dx = \ln x(x\ln x - x) - \int\frac{x\ln x - x}{x}\ dx$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x\ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2 \pm \beta x\ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

....

It is possible to do $\int (\ln x)^2 \, dx$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 e^u \, du = u^2 e^u - \int 2u e^u \, du, = u^2 e^u - 2ue^u \pm 2e^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2\ln 2$ at least once to an expression of the form $u^2 e^u - \beta ue^u \pm \gamma e^u$ Both M's must have been awarded

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0, \; H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • *H* is not likely to be a quadratic function in *x*	B1	3.5b
		(1)	
		(9 marks)	
	Notes		

....

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)

B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.

The first M is for the completed square form of the quadratic showing a maximum at $x = 90$

So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with

$a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x = 90$ at $H_{max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **or** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **and** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = ..., c = ...$	dM1	3.1b
	$H = -\frac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30\,\text{m}$	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\frac{1}{300}(x-90)^2 + 30 \Rightarrow x = ...$	M1	3.4

	$\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120)+b$ leads to $H = -\frac{1}{300}(x-60)(x-120)+27$

Question	Scheme	Marks	AOs
13	$(x-3)^2 + y^2 = \left(\frac{t^2+5}{t^2+1} - 3\right)^2 + \left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$= \frac{(2-2t^2)^2 + 16t^2}{(t^2+1)^2} = \frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2} = \frac{4(t^2+1)^2}{(t^2+1)^2} = 4*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.
dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.
A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x = \frac{t^2+5}{t^2+1} \Rightarrow xt^2 + x = t^2 + 5 \Rightarrow t^2 = \frac{5-x}{x-1}$ $y = \frac{4t}{t^2+1} \Rightarrow y^2 = \frac{16t^2}{(t^2+1)^2} = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2 = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2} = 16\left(\frac{5-x}{x-1}\right) \times \left(\frac{(x-1)}{5-x+x-1}\right)^2 \Rightarrow y^2 = (5-x)(x-1)$	dM1	1.1b
	$y^2 = (5-x)(x-1) \Rightarrow y^2 = 6x - x^2 - 5$ $\Rightarrow y^2 = 4 - (x-3)^2$ or other intermediate step $\Rightarrow (x-3)^2 + y^2 = 4*$	A1*	2.1
		(3)	

(3 marks)
Notes

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.

Other methods exist which also lead to an appropriate equation. E.g using $t = \dfrac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
14	$y = \dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx} = \dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$= \dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$= \dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
			(4 marks)
	Notes		

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y = \dfrac{t^2-4}{2+t} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt}\times\dfrac{dt}{dx} = \dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
14	$y = \dfrac{x-4}{2+\sqrt{x}} \Rightarrow y = \dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}} = \sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
			(4 marks)

Notes

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x} \Rightarrow y=\dfrac{t^2-4}{t+2} \Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n\in\mathbb{N}$ then $(n+1)^3>3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=\ldots$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of n = 1, 2, 3 and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say $n=0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for n =1, 2, 3 and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and **states** even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3 - 12p^2 + 6p + 4 = 2(4p^3 - 6p^2 + 3p + 2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if $m^3 + 5$ is odd then m is even"
- "this is contradiction, so proven."
- "So if $m^3 + 5$ is odd them m is even"