AS proof (exhaustion, deduction and counterexample)

Starter

1. Write down the algebraic form of:
   (a) Two consecutive even numbers.
   (b) Any two even numbers.
   (c) Two consecutive odd numbers.
   (d) Any two odd numbers.

2. Prove that the product of an even and an odd number is even.

Notes

The AS course covered three different types of proof: by exhaustion, by deduction and by counterexample.

Proof by exhaustion

With ***proof by exhaustion***, check all of the values under consideration and if all of them work, the proof is complete.

E.g. 1 Prove that $p^2 + 1$ is not divisible by 3, where $p \in \mathbb{Z}$ and $6 \leq p \leq 10$.

Proof by deduction

Proof by deduction often uses algebra to prove the statement — it is sometimes called ***direct proof***.

E.g. 2 Prove that the sum of any six consecutive integers is divisible by 3 but not by 6.

Disproof by counterexample

Disproof by counterexample simply requires one example that does not work in order to disprove the statement.

E.g. 3 The lowest common multiple of x and y, where $x > 0$ and $y > 0$, is xy. Prove or disprove this statement.

E.g. 4 Prove that the statement "for all positive integers, $n^2 + n + 11$ is prime" is untrue.

Video: Proof by exhaustion and deduction
Proof by counterexample

Exam question: Proof by counterexample

Exercise

p2 1A Qu 1-10, (11-12 red)

Summary

With ***proof by exhaustion***, check all of the values under consideration and if all of them work, the proof is complete.
Proof by deduction often uses algebra to prove the statement — it is sometimes called ***direct proof***.
Disproof by counterexample simply requires one example that does not work in order to disprove the statement.