

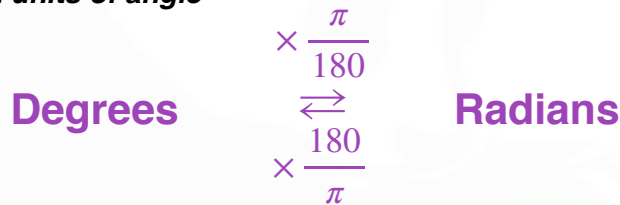
## Arcs and sectors

### Starter

- Given that  $1^c = \frac{180^\circ}{\pi} \approx 57.3^\circ$ :
  - how do we convert from radians to degrees?
  - how do we convert from degrees to radians?
- Convert these angles to radians: (a)  $45^\circ$  (b)  $60^\circ$
- Convert these angles to degrees: (a)  $\frac{\pi}{6}$  (b)  $2\pi$ .

### Notes

#### Converting between units of angle



#### Formulae for length of arc and area of sector in terms of an angle in radians

**E.g. 1** From GCSE, the length of arc,  $s$ , for a sector whose radius is  $r$  and whose angle subtended at the centre is  $\theta^\circ$ , where  $\theta$  is measured in degrees is given by the formula:

$$s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$$

By converting the angle to radians, find a formula for the length of arc when the angle is given in radians.

**Working:** Degrees to radians:  $\times \frac{\pi}{180} \Rightarrow \theta^\circ \times \frac{\pi}{180} = \theta^c$

$$\theta^\circ = \theta^c \times \frac{180}{\pi}$$

Substitute in  $s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ :  $s = \frac{\theta^c \times \frac{180}{\pi}}{360^\circ} \times 2\pi r$

$$s = \frac{180\theta^c}{\pi 360^\circ} \times 2\pi r$$

$$s = r\theta^c$$

**E.g. 2** Using a similar method to **E.g. 1** and utilising the formula  $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$ , find a formula for area of sector, when the angle is measured in radians.

These formulae are not given in the formula booklets:

Length of arc,  $s = r\theta^c$   
Area of sector,  $A = \frac{1}{2}r^2\theta^c$

Remember that with these formulae the **angle must be in radians**.

**E.g. 3** Find the radius of the sector whose arc length is 8.5 cm and where the angle subtended at the centre is  $\frac{3\pi}{4}$ .

**Working:** Using  $s = r\theta^c$ :  $8.5 = r \times \frac{3\pi}{4} \Rightarrow r = \frac{34}{3}$  cm  
The radius of the sector is  $\frac{34}{3\pi} \approx 3.61$  cm

**E.g. 4** Find the area of a sector whose radius is 5 cm and where the angle subtended at the centre is  $\frac{2\pi}{5}$ .

**E.g. 5** Find the perimeter of the sector whose angle subtended at the centre is  $\frac{5\pi}{8}$  and whose area is  $20\pi$  cm<sup>2</sup>. Give your answer in terms of  $\pi$ .

**Video:** [Arcs, sectors and segments](#)

[Arcs, sectors and segments EQ](#)

[Solutions to Starter and E.g.s](#)

### Exercise

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### Summary

Length of arc,  $s = r\theta^c$  — not in the formula booklet

Area of sector,  $A = \frac{1}{2}r^2\theta^c$  — not in the formula booklet

The **angle must be in radians**.