Areas between Curves

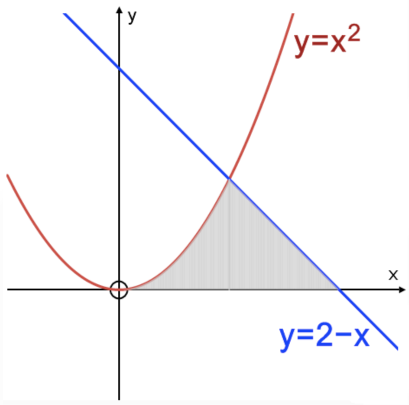
Starter

1. (Review of last lesson)

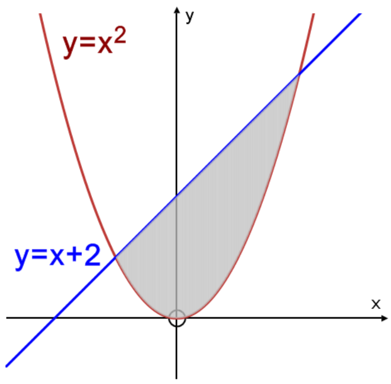
   The surface area of a sphere, S, is expanding at a rate of 9 cm^2 s^{-1}. Find the exact rate of increase of the radius, r, of the sphere when radius is 7 cm.

2. Find the area of the shaded part:

(a)



(b)



Notes

Success Criteria — area between two curves

1. Find the x−coordinates of the points of intersection, say $x = a$ and $x = b$.
2. Area $= \left| \int_a^b f(x)dx - \int_a^b g(x)dx \right| = \left| \int_a^b \big(f(x) - g(x)\big)dx \right|$

Does this formula still work when part of the area is under the x−axis?

Yes

N.B. It is a good idea to sketch the curves to get a visual idea of the area being calculated.

E.g. 1 Find the area between:

(a) the curve $y = x^3$ and the line $y = 4x$

(b) the curves $y = \sin x + 1$ and $y = \cos x + 1$ for $0 \leq x \leq 2\pi$.

Working: (a) To find points of intersection, solve $x^3 = 4x$

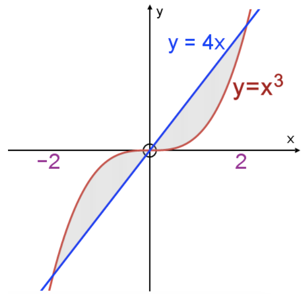
$\therefore x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0$

$x(x-2)(x+2) = 0$

$\Rightarrow x = -2, x = 0$ and $x = 2$

The line $y = 4x$ is above the curve $y = x^3$.

Since areas are symmetrical, find the area from $x = 0$ to $x = 2$ and then double it.



$$\therefore \text{Area} = 2\left(\int_0^2 4x\,dx - \int_0^2 x^3\,dx\right)$$

$$= 2\left[2x^2 - \frac{1}{4}x^4\right]_0^2$$

N.B. The integrals can be combined since the limits are the same.

$$= 2\left(2 \times 2^2 - \frac{1}{4} \times 2^4 - 0\right)$$

Area between the curves $= 8$

E.g. 2 The area between the graphs $y = x^2$ and $y = ax$ is 36, where a is a constant and $a > 0$. Find the value of a.

Video A: Area between 2 curves

Video B: Area between 2 curves

Solutions to Starter and E.g.s

Exercise

p274 12F Qu 1i, 3-6, 9, 11 (10)

Summary

Area between two curves

1. Find the $x-$coordinates of the points of intersection, say $x = a$ and $x = b$.
2. $\text{Area} = \left|\int_a^b f(x)dx - \int_a^b g(x)dx\right| = \left|\int_a^b \big(f(x) - g(x)\big)dx\right|$