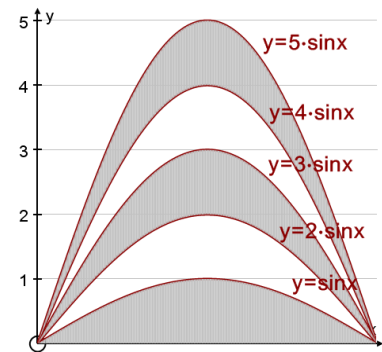


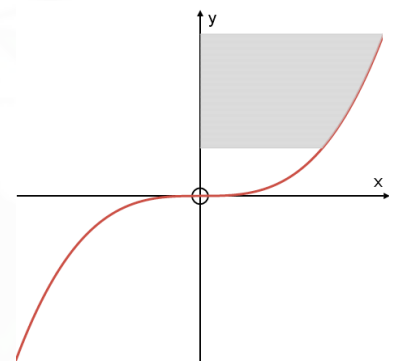
Areas between Curves and the y -axis

Starter

1. **(Review of last lesson)** A company has designed a logo based on multiples of the sine curve as shown. Calculate the total area of the grey sections of the logo.



2. If the area between a curve and the x -axis is $\int_{x_1}^{x_2} y dx$, write down a formula for the area between a curve and the y -axis.
3. Find the area of the region bounded by parts of the y -axis, the curve $y = x^3$ and the lines $y = 8$ and $y = 27$.



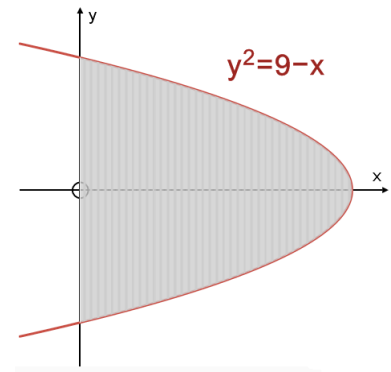
Notes

Area between curve and the x -axis = $\int_{x_1}^{x_2} y dx$

Area between curve and the y -axis = $\int_{y_1}^{y_2} x dy$, where y_1 and y_2 are the y -values for the limits.

Sometime it is possible to find the area between a curve and the y -axis by finding the area of a rectangle and subtracting the area between the curve and the x -axis.

E.g. 1 The graph shows part of the graph of $y^2 = 9 - x$ for which $x > 0$. Find the area between this curve and the y -axis.



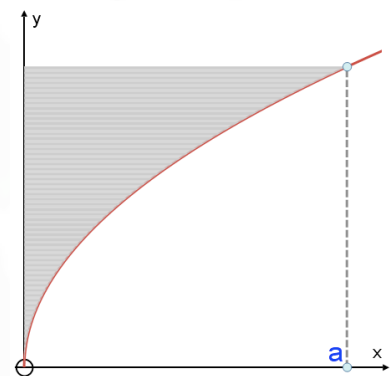
Working: To find the limits find the points where the curve $y^2 = 9 - x$ meets the y -axis:
 y -axis $\equiv x = 0$
 Solve $y^2 = 9 \Rightarrow y = \pm 3$
 Rearrange $y^2 = 9 - x$ to make x the subject:
 $x = 9 - y^2$

$$\begin{aligned} \text{Shaded area} &= \int_{-3}^3 (9 - y^2) dy \\ &= \left[9y - \frac{1}{3}y^3 \right]_{-3}^3 \\ &= \left(27 - \frac{1}{3} \times 27 \right) - \left(-27 - \frac{1}{3} \times (-27) \right) \\ &= 36 \end{aligned}$$

N.B. The area could also be found by doubling the area from $y = 0$ to

$$y = 3 \text{ i.e. } \int_{-3}^3 (9 - y^2) dy = 2 \int_0^3 (9 - y^2) dy$$

E.g. 2 The diagram shows the graph of $y = \sqrt{x}$. Given that the area of the shaded region is 72, find the value of a .



Video: [Area between curve and y-axis](#)

[Solutions to Starter and E.g.s](#)

Exercise

p275 Ex 12F Qu 2, 7, 8, 12

Summary

Area between curve and the x -axis $= \int_{x_1}^{x_2} y dx.$

Area between curve and the y -axis $= \int_{y_1}^{y_2} x dy.$