

Arithmetic Sequences

Starter

- (Review of last lesson)** Write down the terms of $\sum_{r=-2}^3 (r^2 + 1)$ and hence find its value.
- (Review of last lesson)** Express the series $-25 + 36 - 49 + 64$ in sigma notation.

Notes

An **arithmetic progression** (AP) is when successive terms are found by adding (or subtracting) the same number. Or, the difference between successive terms is always the same. This difference is called the **common difference**. The common difference can be positive or negative.

E.g. 2, 5, 8, 11, ... common difference = $5 - 2 = 8 - 5 = 11 - 8 = 3$;
7, 1, -5, ... common difference is -6 .

- E.g. 1** (a) Given that the first term of a general AP is a and the common difference is d , write down the first 4 terms.
(b) Hence express the n -th term, u_n , in terms of a , d and n .

Working: (a) $a, a + d, a + 2d, a + 3d, \dots$
(b) The coefficient of d is always 1 less than then "th term"
i.e. 2nd term has $1d$, 3rd term has $2d$ etc.
Formula for n -th term is $u_n = a + (n - 1)d$

The formula for n -th term of an arithmetic progression is:

$$u_n = a + (n - 1)d$$

N.B. $d = u_2 - u_1$ $d = u_3 - u_2$ $d = u_4 - u_3$ $d = u_{n+1} - u_n$

- E.g. 2** Find the formula for the n -th term for the sequence 13, 15, 17, 19,...

- E.g. 3** Find the number of terms in the sequence 10, 7, 4, ..., -50.

Working: $a = 10$ and $d = 7 - 10 = -3$
 $u_n = a + (n - 1)d \Rightarrow u_n = 10 + (n - 1) \times (-3) = 13 - 3n$
-50 is the last term so $13 - 3n = -50 \Rightarrow 63 = 3n \therefore n = 21$
There are 21 terms in the sequence.

- E.g. 4** The 5th term of an AP is 7 and the 9th term is 43. Find the first term and the common difference.

- E.g. 5*** In an arithmetic sequence the first 3 terms are $\ln x$, $\ln(x + 8)$ and $\ln(x + 48)$. Find the value of x and the next term in the sequence.

Hint: You will need the laws of logs.

Video: [Arithmetic progressions](#)
Video: [Finding a and d given two terms](#)

[Solutions to Starter and E.g.s](#)

Exercise

p72 4C Qu 1-7

Summary

Arithmetic progression: $a, a + d, a + 2d, a + 3d, \dots$

Formula for n -th term: $u_n = a + (n - 1)d$

a = first term d = common difference

$$d = u_2 - u_1$$

$$d = u_3 - u_2$$

$$d = u_4 - u_3$$

$$d = u_{n+1} - u_n$$