Arithmetic Series

Starter

1. **(Review of last lesson)**
Find the number of terms in the sequence 72, 64, 56, 48, …, −288.

2. **(Review of last lesson)** In an arithmetic progression, $u_3 = 15$ and $u_{26} = 84$. Find the value of k if find $u_k = 66$.

3. Find the sum of the first 100 positive integers

Notes

Series is when the terms of a sequence are added together. What is the sum of an arithmetic series?

E.g. 1 Employ the same method used in question 3 of the starter find the sum of the series:

$$S_n = a + a+d + a+2d + \ldots + a+(n-3)d + a+(n-2)d + a+(n-1)d$$

Hint: Write the series out twice — once with the terms in the right order and the 2nd time with the order of the terms reversed.

Working:

Write the series out twice

$$S_n = a + a+d + a+2d + \ldots + a+(n-3)d + a+(n-2)d + a+(n-1)d$$

$$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \ldots + a+2d + a+d + a$$

The sum of the first term and the last is equal to the sum of the second and the penultimate terms etc.

Adding the two series together gives

$$2S_n = 2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d + \ldots + 2a+(n-1)d$$

There n terms, each of $2a+(n-1)d$.

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$ — the formula for the sum of the 1st n terms of an arithmetic progression

N.B. Write out the value of a, d and n before substituting into the formula.

E.g. 2 Find the sum of the 1st 14 terms of 2 + 5 + 8 + …

E.g. 3 The 7th term of an AP is −14 and the sum of the first 7 terms is −35. Find the first term and the common difference.

E.g. 4 How many terms are needed for the sum of the series 1 + 5 + 9 + … to equal 231?

E.g. 5 Given that the last term of a sequence is $l = a + (n-1)d$, write S_n in terms of a, l and n.

Video: Sum of arithmetic series

Arithmetic progressions EQ

Solutions to Starter and E.g.s

Exercise
p74 4D Qu 1i, 2i, 3-16

Summary
Arithmetic progression: $a, a+d, a+2d, a+3d, \ldots.$
Formula for $n-$th term: $u_n = a + (n-1)d$
a = first term d = common difference
$d = u_2 - u_1$ $d = u_3 - u_2$ $d = u_4 - u_3$ $d = u_{n+1} - u_n$
Sum to n terms: $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$