Binomial expansion of compound expressions

Starter

1. **(Review of last lesson)** Find the first ***three*** terms in the expansion of $\dfrac{1}{\sqrt[3]{8-5x}}$ and state the range of values for which it is valid.

2. Given that the coefficient of x^3 in the expansion of $\dfrac{1}{(1+ax)^3}$ is -2160, find a.

Notes

Questions can also include the expansion of more than one bracket.

When a binomial (a polynomial with two terms) multiplies a trinomial (a polynomial with three terms), it is usually a good idea to multiply the trinomial by each term of the binomial in turn.

E.g.
$$\left(x+2\right)\left(1+\frac{3}{5}x-\frac{4}{7}x^2\right) = x+\frac{3}{5}x^2-\frac{4}{7}x^3+2+\frac{6}{5}x-\frac{8}{7}x^2$$
$$= 2+\frac{11}{5}x-\frac{19}{35}x^2-\frac{4}{7}x^3$$

E.g. 1 Find in ascending powers of x, up to and including the x^2 term, the expansion of $\dfrac{x+1}{\sqrt{1-x}}$, stating the values of x for which the expansion is valid.

Working:
$$\frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$$
$$(1-x)^{-\frac{1}{2}} = 1-\frac{1}{2}(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2}(-x)^2+\ldots$$
$$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots$$
$$\frac{x+1}{\sqrt{1-x}} = \left(x+1\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots\right)$$
$$= x+\frac{1}{2}x^2+1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots \quad \textit{ignoring terms above } x^2$$
$$= 1+\frac{3}{2}x+\frac{7}{8}x^2+\ldots$$

The expansion is valid for $-1 < x < 1$

E.g. 2 (a) Find the first three terms in the expansion of $\dfrac{\sqrt{1+2x}}{\sqrt{1-4x}}$ in ascending powers of x.

(b) State the values of x for which the expansion is valid.

(c) By substituting $x = 0.01$ in your expansion, find an approximation for $\sqrt{17}$.

Exercise
p114 6B Qu 1-4, (10-11 red)

Partial fractions and binomial expansion
Algebraic fractions need to be expressed as partial fractions before they can be expanded.

E.g. 3 Find the first three terms in ascending powers of x in the expansion of $\dfrac{5x+7}{(x+1)(x+2)}$, stating the values of x for which this is valid.

Working:
$$\frac{5x+7}{(x+1)(x+2)} \equiv \frac{A}{x+1} + \frac{B}{x+2}$$

Multiply by $(x+1)(x+2)$***:***
$$5x+7 \equiv A(x+2) + B(x+1)$$

Let $x = -2$: $\quad -3 = -B \quad \therefore B = 3$

Let $x = -1$: $\quad 2 = A$

So $$\frac{5x+7}{(x+1)(x+2)} \equiv \frac{2}{x+1} + \frac{3}{x+2}$$

$$\frac{5x+7}{(x+1)(x+2)} \equiv \frac{2}{1+x} + \frac{3}{2+x}$$
$$\equiv \frac{2}{1+x} + \frac{3}{2\left(1+\frac{x}{2}\right)}$$
$$\equiv 2(1+x)^{-1} + \frac{3}{2}\left(1+\frac{x}{2}\right)^{-1}$$

$$2(1+x)^{-1} = 2\left(1 - x + \frac{(-1)(-2)}{1\times 2}x^2 + \ldots\right)$$
$$= 2\left(1 - x + x^2 + \ldots\right)$$

$$\frac{3}{2}\left(1+\frac{x}{2}\right)^{-1} = \frac{3}{2}\left(1 - 1\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{1\times 2}\left(\frac{x}{2}\right)^2 + \ldots\right)$$
$$= \frac{3}{2}\left(1 - \frac{1}{2}x + \frac{1}{4}x^2 + \ldots\right)$$

$$\frac{5x+7}{(x+1)(x+2)} = 2\left(1 - x + x^2 + \ldots\right) + \frac{3}{2}\left(1 - \frac{1}{2}x + \frac{1}{4}x^2 + \ldots\right)$$
$$= \frac{7}{2} - \frac{11}{4}x + \frac{19}{8}x^2 + \ldots$$

$2(1+x)^{-1}$ is valid for $-1 < x < 1$

$\frac{3}{2}\left(1+\frac{x}{2}\right)^{-1}$ is valid for $-1 < \frac{x}{2} < 1$ i.e. $-2 < x < 2$

Therefore, the expansion of $\dfrac{5x+7}{(x+1)(x+2)}$ is valid for $-1 < x < 1$.

E.g. 4 By express $\dfrac{x+7}{(x-1)(x+3)}$ as partial fractions, find its expansion in ascending powers of x up until the term in x^2.

Video: General binomial expansion (from 1:19:51)

Partial fractions and the binomial expansion EQ

Solutions to Starter and E.g.s

Exercise
p114 6B Qu 5-8, (9 red)

Summary
When a binomial (a polynomial with two terms) multiplies a trinomial (a polynomial with three terms), it is usually a good idea to multiply the trinomial by each term of the binomial in turn.

Algebraic fractions need to be expressed as partial fractions before they can be expanded.